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Multiple scattering of an obliquely incident plane acoustic wave from a grating of immersed cylindrical shells

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ABSTRACT

The study of the interaction of acoustic waves with cylindrical structures has numerous applications including the ultrasonic nondestructive testing of materials. In this paper, using a new mathematical model presented for the scattering of obliquely incident plane acoustic waves from a grating of immersed cylindrical shells, a detailed study of the resonant interaction of A-wave resonances originating from the shells is conducted. The nature of A-wave resonances and the effect of center-to-center distance of the shells on these resonances are examined. It is observed that this resonant interaction not only results in the splitting of A-wave resonances, but also causes an increase in resonance amplitudes. This interaction phenomenon is not seen in Rayleigh, whispering gallery and guided wave resonances. It is also shown that increasing the angle of wave incidence to the grating weakens the A-wave resonant interactions. The numerical results obtained from the mathematical model are compared to experimental results available in the literature for gratings composed of two and three aluminum shells. The numerical results are in very good agreement with their experimental counterparts.

1. Introduction

Multiple scattering of ultrasonic waves from a grating of immersed cylinders has numerous applications, such as nondestructive evaluation (NDE) of heat exchangers and characterization of biological tissues. Regularly spaced arrays of tubes immersed in a fluid are commonly found in conventional boilers, heat exchangers and fuel rod assemblies of nuclear reactors. Nondestructive evaluation and fault detection in such structures is important due to safety implications and the high cost of plant downtime.

Early studies of wave scattering from single solid cylinders, conducted by Faran [1], dealt with compression waves normally incident on a submerged infinite homogeneous elastic isotropic rod. Junger developed the theory of acoustic wave scattering from immersed cylindrical shells in 1952 [2]. Resonance scattering theory (RST), developed in 1978 by Uberall and coworkers, presented a new tool for analyzing scattering problems. RST has been applied to various problems including the acoustic wave scattering from immersed fluid-filled elastic cylindrical shells [3]. According to the terminology used in RST, the resonances corresponding to various wave modes are labeled by two indices *n* and *l*. The first index, *n*, defines the ordinal number of the resonance and the second one defines its family. In this terminology, $l = 1$ corresponds to resonances characterized by pseudo-Rayleigh waves and $l = 2, 3, \ldots$ designates resonances featuring whispering gallery (WG) waves.

Talmant et al. [4] used RST for studying the scattering of acoustic waves from thin air-filled immersed aluminum shells. Their measurements showed families of resonances generated by $l = 0$ type waves; indicating the existence of a type of fluid-borne wave around the shell. They compared this type of wave with fluid-borne waves generated on a plate bounded by a fluid on one side and by vacuum on the other side (A-wave). Izbicki et al. [5] showed that if the shell is thin, the waves of the $l = 0$ family behave like the anti-symmetrical Scholte–Stoneley waves which are generated at a fluid–shell interface.

These resonances have been called, according to different authors, $l = 0$, A-waves, or Scholte–Stoneley resonances [6]. Studies have shown that the energy of the $l = 0$ family is primarily located in the surrounding fluid (contrary to $l \neq 0$ modes which are internal resonances) and therefore, they can be labeled as external resonances [6]. Furthermore, Veksler et al. [7] showed that an A-wave has a very significant internal component at low frequencies and becomes more external as the frequency is increased. Moreover, it has been shown that for a cylindrical shell with specific thickness and material properties, these resonances are only seen within a limited frequency band [8].

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The more complex problem involving the analytic calculation of the acoustic field scattered by an immersed grating of infinitely long elastic cylinders could be tackled by using Graf's addition theorem [9,10]. Twersky [11,12] and Dumery [13] studied the general theory of diffraction of acoustic and electromagnetic waves by arbitrary grids in planar or circular configurations. Young and Bertrand [14] presented a simple formulation for the scattered pressure field from two rigid cylinders followed by numerical calculations. Acoustic wave scattering from gratings of obstacles placed in a fluid was considered by Audoly and Dumery [15,16] with applications to acoustic barriers. The formulation works for a finite number of obstacles as well as for the case of an infinite planar grating and takes into account the interactions between different elements.

Further developments were carried out including waves scattered by infinite gratings [17,18], sound propagation and oblique sound transmission in bundles of periodically arranged tubes [19,20], multiple scattering in dense media [21,22], and propagation of a single wave mode (longitudinal or transverse) in a fiber-reinforced composite [23,24]. Guenneau et al. [25] presented a new method for analysis of electromagnetic and elastodynamic waves propagating through a periodic array of cylindrical channels at oblique incidence. Kheddioui et al. [26,27]studied the resonance scattering by two isotropic cylindrical shells immersed in water. Their solution method was similar to that used by Young and Bertrand [14] for solving the scattering problem from rigid cylindrical shells.

Multiple scattering of acoustic waves from a grating of two immersed cylindrical shells was studied theoretically and experimentally by Lethuillier et al. [28,29]. They considered a linear array of N cylindrical shells under normal incidence based on the T-matrix formulation [30]. Their results indicate that for closely-spaced cylindrical shells, a resonant interaction takes place among adjacent cylinders such that each resonance of the Scholte–Stoneley wave is split into two parts at low frequencies. Bas et al. [31] showed that this phenomenon is also seen in the case of a triangular grating of immersed cylindrical shells. They showed that this behavior is not a universal phenomenon, but is only seen at certain angles of incidence.

Robert et al. [32] considered multiple scattering by a finite number of closely-spaced cylindrical cavities (both empty and fluid-filled) embedded in an elastic matrix. They showed that, with the exception of a few narrow resonances, the single fluid-filled cavity resonances do not display the splitting phenomenon seen for immersed elastic scatterers.

An alternative approach based on group theory was developed by Decanini et al. [33] for studying the resonances of two adjacent elastic solid rods. In this theory, the resonances can be classified according to their symmetry properties. This method significantly simplifies the numerical treatment of the problem. Resonance frequencies are determined in the complex frequency plane, and a partial algebraic classification of resonances is obtained corresponding to various boundary conditions.

In this paper, using a new mathematical model presented for the scattering of a plane acoustic wave from a grating of immersed cylindrical shells, a detailed study of the resonant interaction of A-wave resonances originating from the shells is conducted. While in most previous models, the incident wave is only considered to be normal to the cylinders' axes, in the new model, the wave can be incident on the grating at any desired angle. This can be considered as incorporating a third dimension into the model. Experimental results for gratings composed of two and three aluminum shells are used to verify the proposed mathematical model. The nature of A-wave resonances and the effect of center-to-center distance of the shells on these resonances are examined.

2. Mathematical model

An infinite plane acoustic wave of frequency $\omega/2\pi$ incident on a grating of N submerged, infinitely long isotropic cylindrical shells, with inner radius a and outer radius b aligned in one row is considered. The wave vector $\vec{k}$ makes an angle α with respect to the axis of each cylinder, with an azimuthal angle β with respect to the grating as shown in Fig. 1. A separate coordinate system (r_c, θ_c, z) for each cylinder has the z direction parallel to the cylinder axis, with the x-axis (corresponding to $\theta_c = 0$) of each coordinate system coincident with the direction of the incident wave. For an arbitrary cylinder 'c' in the grating, the incident plane wave of amplitude $p_\circ^c$ is denoted as p_i^c when expressed in the (r_c, θ_c, z) polar coordinate system as follows

$$p_i^c = p_\circ^c e^{i(\vec{k}\cdot\vec{r}_c - \omega t)}, \quad c = 1, 2, 3, \ldots, N \tag{1}$$

Applying a series expansion to Eq. (1) for each coordinate system, yields [8],

$$\begin{aligned} p_i^c &= \sum_{n=0}^{\infty} \varepsilon_n p_\circ^c i^n J_n(k_\perp r_c) \cos n\theta_c e^{i(k_z z - \omega t)} \\ &= \sum_{n=-\infty}^{\infty} p_\circ^c i^n J_n(k_\perp r_c) e^{in\theta_c} e^{i(k_z z - \omega t)} \end{aligned} \tag{2}$$

where k is the wave number in water, $k_\perp = k\cos\alpha$, $k_z = k\sin\alpha$, ε_n is the Neumann factor ($\varepsilon_n = 1$ for $n = 0$, and $\varepsilon_n = 2$ for $n > 0$), and J_n are Bessel functions of the first kind of order n. Eqs. (1) and (2) express the incident wave in N local coordinate systems.

2.1. Local and global coordinate systems

In multiple scattering problems, the formulation is usually expressed in a number of local coordinate systems positioned on individual cylinders of the grating. Moreover, it is a common practice to transfer the wave fields from a number of local coordinate systems to one global coordinate system [14,23,30,31]. Consequently, the incident and scattered wave fields are expressed in the global coordinate system. As shown in Fig. 2, for transferring the local coordinate system from cylinder 'c' to a global coordinate system placed on the center of cylinder 1, we have

$$\vec{r}_c = \vec{r}_{c1} + \vec{r}_1 \tag{3}$$

while,

$$\vec{r}_1 = r_1(\cos\theta_1 \hat{e}_x + \sin\theta_1 \hat{e}_y) \text{ and } \vec{r}_{c1} = r_{c1}\hat{e}_y \tag{4}$$

Substituting Eq. (4) into Eq. (3) gives

$$\vec{r}_c = r_1\cos\theta_1 \hat{e}_x + (r_{c1} + r_1\sin\theta_1)\hat{e}_y \tag{5}$$

For a normally incident wave on the grating, the incident wave vector is defined as

$$\vec{k} = k(\cos\beta \hat{e}_x + \sin\beta \hat{e}_y) \tag{6}$$

and therefore, using Eqs. (5) and (6) we have

$$\vec{k}\cdot\vec{r}_c = r_1 k\cos(\theta_1 - \beta) + r_{c1} k \sin\beta \tag{7}$$

Using Eq. (7), we can write

$$\exp(i\vec{k}\cdot\vec{r}_c) = \exp(ir_1 k\cos(\theta_1 - \beta)) \cdot \exp(ir_{c1} k\sin\beta) \tag{8}$$

or,

$$\exp(i\vec{k}\cdot\vec{r}_c) = \exp(ir_{c1} k\sin\beta) \sum_{n=0}^{\infty} \varepsilon_n i^n J_n(kr_1)\cos(n(\theta_1 - \beta)) \tag{9}$$

Substituting Eq. (9) in Eq. (1) for cylinder 'c' gives