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On the Fisher information and design of a flexible progressive censored experiment

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ABSTRACT

We provide simple computational formulas of both expected termination time and Fisher information of the flexible progressive censoring scheme proposed by Bairamov and Parsi (2011). Then, the design and planning of the flexible progressive censoring schemes are discussed with illustrative examples.

1. Introduction

In industrial or clinical experiments, there are many situations in which experimental units or subjects are lost or removed from experimentation before failure. The experimenter may not obtain complete information on failure times for all experimental units. Data obtained from such experiments are called censored data. Censoring can be intentional in which the removal of units or subjects is pre-planned. Saving the total time on the experiment and the cost associated with the failure of the experimental units are the major reasons for pre-planned censoring in life-testing experiments. However, censored data may reduce the efficiency of statistical inference compared to complete data. Therefore, it is desirable to have a censoring scheme to strive for balance between the total time spent for the experiment, number of units failed in the experiment and the efficiency of statistical inference based on the results of the experiment.

Since the conventional Type-I and Type-II censoring schemes (see, for example Balakrishnan and Ng, 2006, Section 2.3) do not allow the removal of units at points other than the terminal point of the experiment, a more general censoring scheme called progressive Type-II right censoring is proposed. Specifically, consider an experiment in which n experimental units are placed on a life-test and the number of observed failures are fixed as $m (m \leq n)$. At the time of the first failure, r_1 units are randomly removed from the remaining $(n-1)$ surviving units. Then, at the time of the second failure, r_2 units from the remaining $(n-2-r_1)$ units are randomly removed. The test continues until the mth failure is observed. For comprehensive overviews and extensive reviews of progressive censoring, one may refer to Balakrishnan and Aggarwala (2000) and Balakrishnan (2007).

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In the past decade, generalizations of the progressive censoring scheme are proposed. For instances, Kundu and Joarder (2006) and Childs et al. (2007) introduced a combination of the Type-I censoring and progressive censoring and proposed a progressive hybrid censoring scheme. Different adaptive progressive censoring schemes (see, for example Ng et al., 2009; Cramer and Iliopoulos, 2010; Bairamov and Parsi, 2011; Kinaci, 2013) which allow the experimenters to choose the number of removal at the ith failure, r_i, by taking into account both the previous numbers of removals $r_1, \ldots, r_{i-1}$ and the previous failure times.

In this paper, we consider an adaptive progressive censoring scheme, called flexible progressive censoring scheme, proposed by Bairamov and Parsi (2011). When designing a censoring scheme of an experiment, the experimenter needs to consider the efficiency of the statistical estimation of the model parameters, which can be measured by the Fisher information, along with the expected termination time of the experiment. In this paper, we provide simple computational formulas for calculating the expected termination time of the experiment and the expected Fisher information for the flexible progressive censoring scheme. This paper is organized as follows. In Section 2, we review the flexible progressive censoring scheme and provide simple formulas for computing the expected termination time of the experiment. Then, we derive the expected Fisher information based on the flexible progressive censoring scheme in Section 3. Experimental planning and design of the flexible progressive censoring scheme are discussed in Section 4. Weibull distributions with different shape parameters are used to illustrate the proposed methodologies. Finally, conclusions are given in Section 5.

2. Flexible progressively censoring scheme and expected termination time of the experiment

Suppose a nonnegative random variable X has an absolutely continuous probability density function (p.d.f.) $f(x;\theta)$ and cumulative distribution function (c.d.f.) $F(x;\theta)$ where θ is a scalar parameter. For the progressive Type-II censoring scheme described in the previous section (see also Balakrishnan and Aggarwala, 2000; Balakrishnan, 2007), we denote the ith progressively censored order statistics (i.e., the time of the ith failure) as $X_{i:m:n}$, $i = 1, \ldots, m-1$ and the progressive censoring scheme as $\mathbf{r} = (r_1, r_2, \ldots, r_m)$. Note that the conventional Type-II censoring scheme is a special case of the progressive Type-II censoring while $r_i = 0$ for all $i = 1, 2, \ldots, m-1$ and $r_m = n-m$. Let $f_{1\cdots m:m:n}(x_1, \ldots, x_m;\theta)$ be the joint probability density function (p.d.f.) of the progressively Type-II censored data, $(X_{1:m:n}, \ldots, X_{m:m:n})$, and let $f_{i:m:n}(x_1, \ldots, x_m;\theta)$ be the marginal p.d.f. of the ith progressively censored order statistic $X_{i:m:n}$. Then, the joint p.d.f. of the progressively Type-II censored data can be expressed as (Balakrishnan and Aggarwala, 2000)

$$f_{1\cdots m:m:n}(x_1, \ldots, x_m;\theta) = \prod_{i=1}^{m} n_{i-1} f(x_i;\theta)\left\{1 - F(x_i;\theta)\right\}^{r_i}, \quad x_1 < \cdots < x_m$$

where $n_i = n - i - \sum_{j=1}^{i} r_j$ with $n_0 = n$ and $n_m = 0$.

In the flexible progressive censoring scheme, the number of removal at each failure is chosen, after comparing each failure time with a pre-specified control time, from two progressive Type-II censoring schemes. Specifically, the number of removal at the ith failure is chosen from r_i^a and r_i^b after comparing the ith failure time with the pre-determined control time t_i. Suppose that $\mathbf{r}_m^a = (r_1^a, \ldots, r_m^a)$ and $\mathbf{r}_m^b = (r_1^b, \ldots, r_m^b)$ are two progressive Type-II censoring schemes where $r_i^a \ge r_i^b$ for $i = 1, \ldots, m-1$. Then, the flexible censoring scheme is $\mathbf{R}_m^F = (R_1^F, \ldots, R_m^F)$,

$$R_i^F = \begin{cases} r_i^a & \text{if } X_{i:m:n} > t_i, \\ r_i^b & \text{if } X_{i:m:n} \le t_i, \end{cases}$$

and $R_m^F = n - m - \sum_{i=1}^{m-1} R_i$, for $i = 1, \ldots, m-1$. Here, R_i^F can be written as $Z_i r_i^a + (1 - Z_i) r_i^b$, where

$$Z_i = \begin{cases} 1 & \text{if } X_{i:m:n} > t_i, \\ 0 & \text{if } X_{i:m:n} \le t_i. \end{cases}$$

This flexible progressive censoring scheme has been generalized by Kinaci (2013) by considering $k(\ge 1)$ t's for each i along with $k+1$ progressive censoring schemes.

Remark 2.1. The adaptive progressive Type-II hybrid censoring scheme proposed by Ng et al. (2009) can be viewed as a special case of the flexible progressive censoring scheme where $t_i = T$, $i = 1, \ldots, m$ and

$$\mathbf{r}_m^a = (r_1, \ldots, r_m),$$
$$\mathbf{r}_m^b = (0, \ldots, 0, n-m).$$

Remark 2.2. In a flexible progressive censoring scheme, one may intend to expedite the experiment if the ith failure comes later than the predetermined time t_i. From the basic properties of order statistics (see, for example David and Nagaraja, 2003, Section 4.4), the larger the number of items in the experiment is, the smaller the expected total test time (Ng and Chan,