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On Hong–Tamer's estimator in nonlinear errors-in-variable regression models

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ABSTRACT

Under some regularity conditions, the paper provides an alternative proof for the revised moment conditions proposed by Hong and Tamer (2003) in the nonlinear least squares regression model, when the covariates are measured with Laplace error. The asymptotic normality of the revised moment estimates is developed. The choice of optimal weight functions is also discussed and a nearly optimal weight function is identified. Moreover, a simulation extrapolation estimation procedure is suggested when the estimating equations based on the revised moment conditions are difficult to solve. Simulation studies are conducted to evaluate the finite performance of the proposed methods.

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1. Introduction

Constructing proper estimating equations is a commonly used and often effective way to find the estimates for the unknown parameters in many econometric models. The estimating equations are usually the sample versions of their population analogs. For example, the parameter of interest, say $\boldsymbol{\beta}$, might be defined by a set of population moment conditions, $Em(\mathbf{X}; \boldsymbol{\beta}) = 0$, where $\mathbf{X} = (X_1, \ldots, X_k)^\tau$ is a k-dimensional random vector, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)^\tau$ is a p-dimensional vector of unknown parameters to be estimated, and $m(\cdot; \cdot)$ is a vector of functions. For any vector or matrix, the superscript τ means transposition. Econometric and statistical literatures are abundant in solving the estimation equations, as well as the discussion of the large sample properties of the resulting estimates. In many real applications, the vector $\mathbf{X}$ may not be observed directly. Instead, a surrogate $\mathbf{Z} = (Z_1, \ldots, Z_k)^\tau$ is available, which related to $\mathbf{X}$ additively through the relationship $\mathbf{Z} = \mathbf{X} + \mathbf{U}$, where $\mathbf{U} = (U_1, \ldots, U_k)^\tau$ is called the measurement error. It is well known that the presence of measurement error often creates some model identification problems. See Fuller (2006) for such examples. To identify the model parameters, we can either impose stronger distributional assumptions on random entities in the model, or seek extra data resources, such as the validation data set and replication measurements. Hong and Tamer (2003) tackled the identifiability problem by assuming the p random variables in $\mathbf{U}$ to be independent and each follows a Laplace distribution with mean 0 and unknown variance. Moreover, under the Laplace measurement error, the moment conditions $Em(\mathbf{X}; \beta) = 0$ can be replaced by the revised moment conditions based on the observed variables only.

To be specific, suppose the measurable function $m(\mathbf{x}; \boldsymbol{\beta})$ satisfies the Assumption 3 in Hong and Tamer (2003). Then follows a Laplace distribution with characteristic function $\phi_{\mathbf{U}}(\mathbf{t}) = \prod_{j=1}^{k}(1 + \sigma_j^2 t_j^2/2)^{-1}$, where $\mathbf{t} = (t_1, \ldots, t_p)^\tau$, then they

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showed that

$$Em(\mathbf{X};\boldsymbol{\beta}) = Em(\mathbf{Z};\boldsymbol{\beta}) + \sum_{l=1}^{k}\left(-\frac{1}{2}\right)^{l}\sum\cdots\sum_{j_1<\cdots<j_l}\sigma_{j_1}^2\cdots\sigma_{j_l}^2 E\frac{\partial^{2l}m(\mathbf{X};\beta)}{\partial X_{j_1}^2\cdots\partial X_{j_l}^2}. \quad (1)$$

If the revised moment conditions are sufficient for identifying σ and β, then the estimating equations based on the sample version of the above revised moment conditions can be employed to derive the estimates and show their consistency and asymptotic normality. However, if the revised moment conditions cannot completely identify the unknown parameters, some identified features can be still consistently estimated by the so called modified moment estimation procedure using some side information about the relative magnitude of the measurement error variances. The methodology cannot be easily extended to normal measurement error model case unless the function m has some simpler structures, such as polynomial or exponential. In fact, the similar differentiation idea was already adopted in the Masry and Rice (1992) when dealing with the deconvolution density estimate. Although Masry and Rice (1992) mainly discuss the normal measurement error case, they did mention that the same technique could be used for Laplace measurement error case.

Hong and Tamer (2003) provided a very nice proof for (1) using the deconvolution relationship between the density functions of **X** and **Z**. Another way of proving (1) is to consistently estimate the left hand side of (1) using the deconvolution kernel density estimate, then show the estimate also converges to the right hand side of (1) in probability. To be specific, let K be a symmetric kernel density function and b_n denote a sequence of bandwidth depending on the sample size n, then a consistent deconvolution kernel estimate for the density function of **X** can be written as

$$\hat{f}(\mathbf{x}) = \frac{1}{nb_n^k}\sum_{j=1}^{n}L_n\left(\frac{\mathbf{x}-\mathbf{Z}_j}{b}\right), \quad (2)$$

where $L_n(\mathbf{x})$ is defined by

$$L_n(\mathbf{x}) = \prod_{l=1}^{k}\frac{1}{2\pi}\int \exp(-\mathbf{i}t_l x_l)\frac{\psi_K(t_l)}{\psi_{U_l}(t_k/b_n)}dt, \quad \mathbf{i}^2=-1,$$

where $\mathbf{x}=(x_1,\ldots,x_k)$, $\mathbf{Z}_j=(Z_{j,1},\ldots,Z_{j,k})^{\tau}$, and for any generic random variable or vector X, ψ_X denotes its characteristic function. If we choose K to be the standard normal density function ϕ, then

$$L_n(\mathbf{x}) = \prod_{l=1}^{k}\left[\phi(x_l) - \frac{\sigma_l^2}{2b_n^2}\phi''(x_l)\right].$$

Accordingly, an estimate of $Em(\mathbf{X};\boldsymbol{\beta})$ can be obtained by directly calculating the expectation of $m(\mathbf{X};\boldsymbol{\beta})$ with respect to $\hat{f}(\mathbf{x})$. In fact

$$\begin{aligned}\widehat{Em(\mathbf{X};\boldsymbol{\beta})} &= \int m(\mathbf{x};\boldsymbol{\beta})\frac{1}{nb_n^k}\sum_{j=1}^{n}L_n\left(\frac{\mathbf{x}-\mathbf{Z}_j}{b_n}\right)d\mathbf{x}\\ &= \frac{1}{nb_n^k}\sum_{j=1}^{n}\int m(\mathbf{x};\beta)\prod_{l=1}^{k}\left[\phi\left(\frac{x_l-Z_{j,l}}{b_n}\right) - \frac{\sigma_l^2}{2b_n^2}\phi''\left(\frac{x_l-Z_{j,l}}{b_n}\right)\right]d\mathbf{x}. \quad (3)\end{aligned}$$

We can show that under some regularity conditions, the right hand side of the above equality is a consistent estimator of the right hand side of the revised moment formula. A sketch of the proof can be found in Section 5.

The significance of Theorem 1 of Hong and Tamer (2003) is demonstrated by the fact that in Laplace measurement error case, the estimating equations based on the true variables, which cannot be used in real application due to the lack of observations for mismeasured variables, can be replaced by estimating equations involving second order partial derivatives of the original estimating functions based on the observed variables. Therefore, one can estimate the unknown parameters just like we are facing a classic estimation problem. Similar phenomenon can be found in nonparametric regression with Laplace measurement error. In fact, Example 2 from Fan and Truong (1993) shows that estimating the regression function is equivalent to estimating up to the second order derivative of the regression function of the response variable on the observed surrogates.

In the nonlinear regression setup, Hong and Tamer (2003) provided explicit revised moment conditions and indicated that these conditions could be justified by using (1), see Example 2 in Hong and Tamer (2003). To be specific, suppose the parametric regression model is $E(Y|X=x)=g(x;\boldsymbol{\beta})$, where Y is a scalar response variable and X is a scalar predictor, g is a known twice differentiable function, and $E[Y^2|X=x]$ is finite. For any measurable function $h(\cdot)$ with finite second moment, possibly vector-valued, we have $E[h(X)(Y-g(X;\boldsymbol{\beta}))]=0$. Then based on (1), the authors claimed that the revised moment conditions are

$$E\left[h(Z)R(Y,Z;\boldsymbol{\beta}) - \frac{\sigma^2}{2}\left(h''(\mathbf{Z})R(Y,Z;\boldsymbol{\beta}) - 2h'(Z)g'(Z;\beta) - h(Z)g''(Z;\boldsymbol{\beta})\right)\right] = 0, \quad (4)$$