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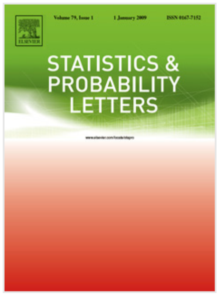

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Mean Square Polynomial Stability of Numerical Solutions to a Class of Stochastic Differential Equations

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Abstract

The exponential stability of numerical methods to stochastic differential equations (SDEs) has been widely studied. In contrast, there are relatively few works on polynomial stability of numerical methods. In this letter, we address the question of reproducing the polynomial decay of a class of SDEs using the Euler–Maruyama method and the backward Euler–Maruyama method. The key technical contribution is based on various estimates involving the gamma function.

Keywords: Polynomial stability, Nonlinear SDEs, Euler-type method, Gamma function, Numerical reproduction

1. Introduction

The stability of stochastic differential equations (SDEs) has been widely studied by many authors, see for instance [20, 11, 16, 10] and references therein. In particular, different decay rates have been widely investigated, for example, exponential stability [13, 18], polynomial stability [15, 1] and general rate [12, 4].

It is natural to ask whether numerical solutions of SDEs preserve stability properties of the original SDEs and in recent years, this question has received quite a lot of attention. See for instance [21, 22, 5] for some of the original ideas in this area. An important feature of these works is that they focus on the exponential stability of the numerical solutions [6, 19, 23, 24]. Briefly speaking, the mean square exponential stability with rate $r_e > 0$ of a numerical solution (see Section 3 for details on definition of numerical solutions) $\{X_k\}_{k=1,2,3,\ldots}$ with step size Δt is defined as

$$\limsup_{k\to\infty} \frac{\log \mathbb{E}|X_k|^2}{k\Delta t} \leq -r_e.$$

As far as we know, there are few papers devoted to the mean square polynomial stability of the numerical solutions. For some numerical solution $\{Y_k\}_{k=1,2,3,\ldots}$, the mean square polynomial stability

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