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Sound insulation of double-leaf walls – Allowing for studs of finite stiffness in a transfer matrix scheme

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ABSTRACT

In a recent paper by the present author the effect of finite structural connections on the sound reduction index of double walls was predicted by modifying a model based on the transfer matrix technique. However, the model did not include any means to account for the flexibility of the studs; they were assumed to be of infinite stiffness. Based on data for the effective stiffness of flexible steel studs, the model is extended to take account of this flexibility. A number of comparisons are performed, mainly with the measured sound reduction index of lightweight double walls with gypsum boards. Cases include walls with cavity filling as well as with empty (air-filled) cavities. In the latter cases, the energy losses of the cavity are simulated using a model of a porous layer with a minute flow resistivity. Predicted results compare favourably with measurement results. It is assumed that different basic types of studs, i.e. other than the TC-type simulated here may successfully be included in the model.

1. Introduction

In a recent paper by Vigran [1], reviving the semi-empirical method by Sharp [2] to account for the effect of structural connections, such as studs or ties, in double walls, the prediction method was based on a transfer matrix scheme. However, only connections of infinite stiffness were accounted for, which for studs in practice implies wooden ones only and not commonly used metal studs of various cross shapes.

It is well known that a large number of prediction methods exits in the literature and Hongisto [3] implemented all together seventeen of these models making a quantitative comparison between models against measurement results taken from a large series of laboratory measurements; see Hongisto et al. [4]. In the type of constructions we shall be concerned with here, double-leaf with structural connections a couple of models were able to predict the effect of flexible studs. However, only the model of Davy [5] gave satisfactory result, a model which has recently been somewhat modified; see Ref. [6]. The model compares favourably with a number of measurement results on plasterboard walls. However, it is depending on a reasonable guess on the size of a so-called stud attenuation factor, which must be based on comparison with measured end results, i.e. the sound reduction index of the walls.

More recently, Legault and Atalla [7] have, using a special double wall system consisting of two aluminium plates with a cavity filled with a fibrous material, compared altogether five models including their own periodic model. Predictions are compared with measurements results in two cases; with C-sections channels connecting the aluminium plates as well as without these connections. Again, Davy's approach gives quite good agreement but cannot, in contrast to the periodic model, predict the finer details in the measured reduction index.

The role of studs in the sound transmission has recently been given broad coverage also by Poblet-Puig et al. [8], outlining procedures to obtain more accurate parameters to be used in prediction models. By numerical modelling, metal studs of altogether six different cross sections were tested in three types of lightweight double walls. The vibration level difference of the two leaves is a useful parameter for testing the effect of different studs on the same wall. However, as it will be dependant also on other variables, such as the material properties of the leaves, an averaged response corrected stiffness of the studs is given, presented as a frequency-dependant translational stiffness of the different types of studs. As opposed to this simulation procedure, one may certainly envisage a direct measurement in line with other methods on resilient elements; see e.g. Ref. [9].

In this paper, however, we shall use the stiffness data of Poblet-Puig et al. [8] to predict the sound reduction index of double walls with flexible studs, modifying the method given in Ref. [1] to account for the finite stiffness of the studs. For completeness, some important aspects of the method are repeated below.

For comparison with measurement we shall use data for double-leaf gypsum plasterboard walls in the cases where the flexibility data from Ref. [8] seems appropriate, i.e. when the dimensions of the studs are comparable. However, the special construction

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used by Legault and Atalla [7] will also be modelled using the present approach.

2. Theory

Following Sharp [2], the diffuse field sound reduction index R of a double-leaf partition with sound bridges (see Fig. 1a), may be expressed as:

$$R = 10\lg\left[\frac{W_i}{W_{2,P} + W_{2,B}}\right]. \tag{1}$$

Here W_i is the incident power and the power radiated from plate 2 is divided into two parts, $W_{2,P}$ and $W_{2,B}$, the power radiated from plate 2 without the bridges and the power radiated due to the action of the bridges, respectively. These two contributions to the radiated power are assumed to be uncorrelated. Eq. (1) may be written as:

$$R = R_P - \Delta R = R_P - 10\lg\left[1 + \frac{W_{2,B}}{W_{2,P}}\right], \tag{2}$$

where R_P is the sound reduction index of the double-leaf partition without the bridges and ΔR is the correction term due to the bridges. It may be shown that the power ratio in the correction term may be expressed as:

$$\frac{W_{2,B}}{W_{2,P}} = n\sigma_B \cdot \left|\frac{v_B}{v_1}\right|^2 \cdot \left\langle\left|\frac{v_1}{v_2}\right|^2\right\rangle = n\sigma_B \cdot \left|\frac{Z_{B1}}{Z_{B1} + Z_{B2}}\right|^2 \cdot \left\langle\left|\frac{v_1}{v_2}\right|^2\right\rangle, \tag{3}$$

where σ_B is the radiation factor of the second plate driven by one of a number n bridges acting over the partition area S. The second term contains the input impedances of the plates seen from the sound bridge, which is given below. Expressions for the radiation factor, both for line- and point-driven plates are given in Ref. [1] and not repeated here.

The last term in Eq. (3) is the squared ratio of the velocities of plates 1 and 2 in the absence of the bridges assuming diffuse sound incidence, the latter indicated by the outer brackets $\langle\rangle$ Sharp [2] gives an approximate expression for this ratio but in the framework of transfer matrices there is no need to do so. For a given angle of incidence we may express this ratio in terms of the resulting transfer matrix comprising the construction without the bridges, i.e. two plates with a cavity partly or wholly filled with a porous layer. Using a thin plate model and representing any porous material as an equivalent fluid we get a 2×2 matrix with components A_{11}, A_{12}, A_{21} and A_{22}, see e.g. Refs. [3,10]. Denoting the squared velocity ratio at a given angle of incidence by r_θ, we get

$$r_\theta = \left|\frac{v_1}{v_2}\right|^2 = |A_{21} \cdot Z_L + A_{22}|^2, \tag{4}$$

where Z_L represents the fluid loading impedance of the surrounding medium (air). Averaging this expression over the incidence angle θ in the same way as when averaging the transmission coefficient to obtain R_P, will give us the last term in Eq. (3). In all examples shown here a 32-point Gaussian integration routine is used with an upper limit of 90°.

Sharp [2] assumed that the bridges were massless and infinitely stiff, which implies e.g. that the studs shown in Fig. 1a should be represented by massive wooden studs and not by the ones illustrated, which is a common shape of metal studs, denoted TC in Ref. [8]. To allow for flexible studs, we may as a thought experiment illustrated in Fig. 1b, simply insert a spring of mechanical impedance Z_s between the stud and plate 1. This will modify the ratio between the velocity v_B of the bridge and the velocity v_1 of plate 1, which for infinitely stiff bridges is given by:

$$\frac{v_B}{v_1} = \frac{Z_{B1}}{Z_{B1} + Z_{B2}}, \tag{5}$$

being represented by the second term in Eq. (3). Introducing the spring of impedance Z_s we get a parallel combination with the impedance Z_{B1} of the input plate resulting in

$$\frac{v_B}{v_1} = \frac{Z_{B1}}{Z_{B1} + Z_{B2}\left(1 + \frac{Z_{B1}}{Z_s}\right)} = \frac{Z_{B1}}{Z_{B1} + Z_{B2}\left(1 - i\omega\frac{Z_{B1}}{s(\omega)}\right)}, \tag{6}$$

where we have represented the impedance of the spring by a frequency-dependant translational stiffness s. As in Ref. [1] a time dependence $\exp(-i\omega t)$ is used. Furthermore, for the plate impedances we shall use the classical expressions for the input impedances of an infinite plate, which for point- and line-drive may be written:

$$Z_{B,\text{point}} = 8\sqrt{mB}$$
$$Z_{B,\text{line}} = \frac{2(1-i)m\omega}{k_p}, \tag{7}$$

where m, B and k_p are the plate mass per unit area, bending stiffness and free surface wavenumber, respectively.

It should be mentioned that other procedures to modify Sharp's expression to include flexible studs are given in the literature, e.g. Gu and Wang [11]. Their method is included in the comparison performed by Hongisto [3] as well as the one by Legault and Atalla [7].

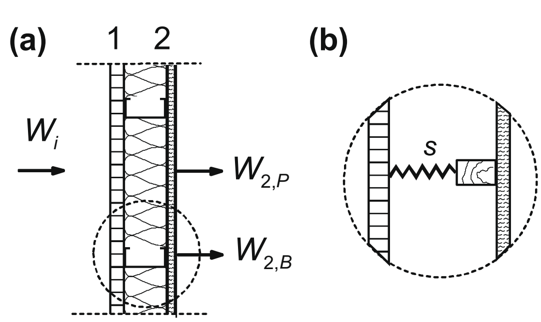

Fig. 1. (a) Sketch of a double wall with studs (TC-type steel studs), (b) representation of a flexible stud (expanded and redrawn detail indicted in 1a).

3. Measured and predicted results

An extensive series of measured results on gypsum board walls is made available by the National Research Council of Canada, see Ref. [12]. Measurements comprise walls with wood studs and steel studs, staggered as well as directly coupling the boards. As for the steel studs, they appear to be of the type TC, mentioned above. Unfortunately, some material data is missing such as the elastic modulus of the boards used but this may be estimated from the surface weight and the coincidence frequency apparent from the measured data.

We shall use the measurement results for several of the walls with steel studs to compare with results using the prediction model outlined above. This necessitates an estimate of the translational stiffness s given in Eq. (6). As mentioned in the introduction we shall use the data given by Poblet-Puig et al. [8] to arrive at an estimate that may be used in our model. It should also be mentioned that data from Ref. [12] is also used by Davy [6] to compare with his model giving reasonably good agreement.

3.1. Stiffness of metal studs

From the calculated vibration level difference of the outer leaves of three different double walls, Poblet-Puig et al. [8] derived