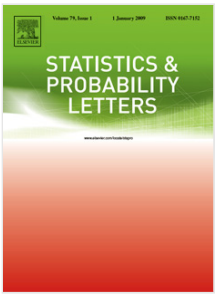

Anticipated backward stochastic differential equations with jumps under the non-Lipschitz condition

Shuheng Tu, Wu Hao

PII: S0167-7152(14)00203-X
DOI: http://dx.doi.org/10.1016/j.spl.2014.06.001
Reference: STAPRO 6993

To appear in: *Statistics and Probability Letters*

Received date: 5 September 2013
Revised date: 29 May 2014
Accepted date: 1 June 2014

Please cite this article as: Tu, S., Hao, W., Anticipated backward stochastic differential equations with jumps under the non-Lipschitz condition. *Statistics and Probability Letters* (2014), http://dx.doi.org/10.1016/j.spl.2014.06.001

ANTICIPATED BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS WITH JUMPS UNDER THE NON-LIPSCHITZ CONDITION. *

SHUHENG TU AND WU HAO*

Abstract. This paper deals with a class of anticipated backward stochastic differential equations with poisson jumps (BSDEJs). We first show that there is a duality between anticipated backward stochastic differential equations with jumps and stochastic differential delay equations with jumps (SDDEJs). Then, we prove the existence and uniqueness of adapted solutions and L^p solutions for such ABSDEJs under the non-Lipschitz conditions as well as a comparison theorem is obtained through constructing some iterative equations which are different from iterative equations in [15].

1. Introduction

The notion of non-linear backward stochastic differential equations (BSDEs) were introduced by Pardoux and Peng [3]. A solution of this equation, associated with a terminal value ξ and a generator or coefficient $f(t,\omega,y,z)$, is a couple of adapted stochastic process $(Y_t,Z_t)_{\{t\in[0,T]\}}$ such that:

$$Y(t)=\xi+\int_t^T f(s,Y(s),Z(s))\mathrm{d}s-\int_t^T Z(s)\mathrm{d}W(s),$$

where W is a d-dimensional standard Brownian motion. This type of nonlinear backward stochastic differential equation was firstly studied by Pardoux and Peng in [3], and they established the existence and uniqueness of adapted solutions under the uniform Lipschitz conditions of f. Since this first result, it has been widely recognized that BSDEs provide a useful framework for formulating a lot of mathematical problems such as used in financial mathematics, optimal control, stochastic games and partial differential equations. Based on the above important applications, specially in the field of Finance, and optimal control, recently, a new classes of BSDEs, called anticipated BSDEs (ABSDEs), were introduced by Peng and Yang [15].

For anticipated BSDEs, we mention the generators include not only the values of solutions of presents but also the future. So ABSDEs may be used in finance. From Theorem 2.1 in [15] we know that there is a perfect duality between stochastic differential delay equations and anticipated BSDEs which can be used in optimal control. Theorem 4.2 in [15] also tells us such a equation has a unique solution under the Lipschitz conditions.

We also mention that, following Peng and Pardoux [3], many papers were devoted to improving the results of Peng and Pardoux [3] by weakening the Lipschitz conditions on coefficients (cf. [[1, 2, 7, 9, 10, 13, 18]). In particular, Hu and Lerner [20] have studied the existence and uniqueness of solutions to BSDEs under the integral-Lipschitz conditions. Furthermore, since BSDEs only driven by Brownian motion can not simulated emergencies, lots of papers studied BSDEs with jumps (cf. [5, 8, 11]) to simulate emergencies. They also obtained the existence and uniqueness of solutions, comparison theorem and some other results.

* Corresponding author : Wu Hao.