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Perturbations and projections of Kalman–Bucy semigroups

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Abstract

We analyse various perturbations and projections of Kalman–Bucy semigroups and Riccati equations. For example, covariance inflation-type perturbations and localisation methods (projections) are common in the ensemble Kalman filtering literature. In the limit of these ensemble methods, the regularised sample covariance tends toward a solution of a perturbed/projected Riccati equation. With this motivation, results are given characterising the error between the nominal and regularised Riccati flows and Kalman–Bucy filtering distributions. New projection-type models are also discussed; e.g. Bose–Mesner projections. These regularisation models are also of interest on their own, and in, e.g., differential games, control of stochastic/jump processes, and robust control.
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1. Introduction

The purpose of this work is to analyse a number of perturbations and projections of Kalman–Bucy [47,16] semigroups and of the associated (matrix differential) Riccati flow.

The prime motivating application for this work is the ensemble Kalman filter (`EnKF`) [32] and the various “regularisation” methods used to ensure well-posedness of the sample covariance

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(e.g. sufficient rank) and to "move" the sample covariance closer (in some sense) to the Riccati flow of the true Kalman filter [47,16]. For example, two common forms of regularisation are covariance inflation-type methods (perturbations) and so-called covariance localisation methods (projections). Covariance inflation is a simple idea that involves adding some positive-definite matrix to the sample covariance in order to increase its rank [7]; i.e. more specifically to account for an under-representation of the true variance due to a potentially inferior sample size. Separately, the idea of covariance localisation involves multiplying (element-wise) the EnKF sample covariance matrix via Schur (or Hadamard) products with certain sparse "masking" matrices with the intent of reducing spurious long-range correlations and increasing the sample covariance rank [44,60]. See [40] for an empirical examination of both types of regularisation. In these two cases, choosing the right inflation or localisation is non-trivial and numerous ideas exist; e.g. [34,35,4,55,5,68,6]. Other related, and/or more subtle, regularisation methods exist and we will cover more general models in more detail in later sections; see also [41,64,3,73,69,45,70,43] for related EnKF methodology.

Note that the total literature on EnKF methodology is too broad to cover adequately here. Results on EnKF convergence are recent (relative to this work) and concern, e.g., weak convergence with sample size [53,58,51], and stability [48,74,75,29,27]. The articles [75,56] concern stability and robustness of the EnKF in the presence of specific inflation and localisation methods. The article [17] studies the behaviour of a stochastic matrix Riccati equation that captures the flow of the sample covariance in a naive EnKF implementation; i.e. its moment behaviour (non-asymptotic bias and variance), convergence and central-limit-type behaviour.

From a purely mathematical vantage, regularisation amounts to studying various projections and perturbations of the "standard" Riccati flow (viz [47,16]). The analytical behaviour of general projections and perturbations are a major focus of this study. We consider a broad class of perturbation model. We consider a particular projection model, and a certain class of localisable/diagonalisable systems adapted to these projections; the details are specified later. New ideas concerning projections relevant to the EnKF are also introduced within this class. Given this analysis, we then study the (nonlinear) Kalman–Bucy diffusion [16] and provide a number of contraction-type convergence results between the corresponding perturbed/projected diffusion and the optimal Kalman–Bucy diffusion. We study convergence in the mean-square sense and also in terms of the law of the diffusion.

While methods in data assimilation and ensemble Kalman filtering are the main drivers of this work, the types of perturbations considered herein are more widely relevant: For example, our analysis captures well those perturbations of the "standard" Riccati flow that arise in, e.g., linear–quadratic differential games [13,59,28], in the control of linear stochastic jump systems [26,2], in certain robust and H^∞ control settings [33,12], etc.; see also the early work of Wonham [78] in linear–quadratic stochastic control. We also highlight the text [1, e.g. Chap. 6] and the references therein. Separately, a specific projected Riccati flow is studied in [21]. Other relevant and related literature considering similar-type projections in estimation theory is given in [66,57]. Going forward, we primarily rely on EnKF motivators, but we emphasise here that the mathematical development is more broadly applicable.

Further introduction, discussion, and background is given in later subsections with a more technical focus.

1.1. Kalman–Bucy diffusions

The notation used throughout this article is introduced later in Section 1.3. However, the set-up in this section is relatively standard. Consider a time homogeneous linear-Gaussian filtering

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