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Representations of max-stable processes via exponential tilting

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Received 25 May 2016; received in revised form 11 October 2017; accepted 21 October 2017
Available online xxxx

Abstract

The recent contribution (Dieker and Mikosch, 2015) obtained representations of max-stable stationary Brown–Resnick process $\zeta_Z(t), t \in \mathbb{R}^d$ with spectral process Z being Gaussian. With motivations from Dieker and Mikosch (2015) we derive for general Z, representations for ζ_Z via exponential tilting of Z. Our findings concern *Dieker–Mikosch representations* of max-stable processes, two-sided extensions of stationary max-stable processes, inf-argmax representation of max-stable distributions, and new formulas for generalised Pickands constants. Our applications include conditions for the stationarity of ζ_Z, a characterisation of Gaussian distributions and an alternative proof of Kabluchko's characterisation of Gaussian processes with stationary increments.

MSC: primary 60G15; secondary 60G70

Keywords: Max-stable process; Spectral tail process; Brown–Resnick stationary; Dieker–Mikosch representation; Inf-argmax representation; Pickands constants; Tilt-shift formula; Extremal index

1. Introduction

A random process $\zeta(t), t \in \mathcal{T}$ is max-stable if all its finite dimensional distributions (fidi's) are max-stable. For simplicity we shall assume hereafter that $\zeta(t)$ has unit Gumbel distribution $e^{-e^{-x}}, x \in \mathbb{R}$ for all $t \in \mathcal{T}$ and shall consider $\mathcal{T} = \mathbb{R}^d$ or $\mathcal{T} = \mathbb{Z}^d, d \geq 1$. In view of [31] any stochastically continuous max-stable process $\zeta(t), t \in \mathcal{T}$ satisfies (below $\stackrel{fdd}{=}$ means equality of

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https://doi.org/10.1016/j.spa.2017.10.003
0304-4149/

Please cite this article in press as: E. Hashorva, Representations of max-stable processes via exponential tilting, Stochastic Processes and their Applications (2017), https://doi.org/10.1016/j.spa.2017.10.003.

all fidi's)

$$\zeta \stackrel{fdd}{=} \zeta_Z,$$

with

$$\zeta_Z(t) = \max_{i\ge 1}\big(P_i + Z_i(t)\big), \quad t \in \mathcal{T}, \tag{1.1}$$

where $Z(t), t \in \mathcal{T}$ is a random process taking values in $[-\infty, \infty)$ with $\mathbb{E}\{e^{Z(t)}\} = 1, t \in \mathcal{T}$ and $\Pi = \sum_{i=1}^{\infty}\varepsilon_{P_i}$ is a Poisson point process (PPP) on $\mathbb{R}$ with intensity $e^{-x}dx$. Further, Z_i's are independent copies of Z being also independent of Π; see for more details [21,37,41,48,52,55,62,63,67,68,71].

We shall refer to ζ_Z as the *associated max-stable* process of Z; commonly Z is referred to as the *spectral process*. For convenience, we shall write Z as

$$Z(t) = B(t) - \ln \mathbb{E}\left\{e^{B(t)}\right\}, \quad t \in \mathcal{T}, \tag{1.2}$$

with $B(t), t \in \mathcal{T}$ a random process satisfying $\mathbb{E}\{e^{B(t)}\} < \infty, t \in \mathcal{T}$. Consequently, $\mathbb{E}\{e^{Z(t)}\} = 1, t \in \mathcal{T}$ implying that the marginal distribution functions (df's) of ζ_Z are unit Gumbel.

One canonical instance is the classical Brown–Resnick construction with B being a centred Gaussian process with covariance function r and thus $2\ln \mathbb{E}\{e^{B(t)}\} = r(t,t) =: \sigma^2(t), t \in \mathcal{T}$. In view of [39] the law of ζ_Z is determined by the incremental variance function $\gamma(s,t) = Var(B(t) - B(s)), s, t \in \mathcal{T}$. This fact can be shown by utilising the *tilted spectral process* $\Xi_h Z, h \in \mathcal{T}$ defined by

$$\Xi_h Z(t) = B(t) - B(h) - \gamma(h,t)/2, \quad t \in \mathcal{T}.$$

The law of $\Xi_h Z$ is uniquely determined by the following conditions: $\Xi_h Z$ is Gaussian, $\Xi_h Z(h) = 0$ almost surely (a.s.) and the incremental variance function of $\Xi_h Z$ is γ. Note that these conditions do not involve σ^2.

Next, setting $Z^{[h]}(t) = B(t) - \sigma^2(t)/2 + r(h,t)$ we have

$$\Xi_h Z(t) = Z^{[h]}(t) - Z^{[h]}(h), \qquad t \in \mathcal{T}. \tag{1.3}$$

In view of Lemma 6.1 $Z^{[h]}$ is the exponential tilt of Z by $Z(h)$ i.e.,

$$\mathbb{P}\{Z^{[h]} \in A\} = \mathbb{E}\left\{e^{Z(h)}\mathbb{I}\{Z \in A\}\right\}, \quad \forall A \in \mathcal{B}(\mathbb{R}^{\mathcal{T}}),$$

where $\mathcal{B}(\mathbb{R}^{\mathcal{T}})$ is the σ-field generated by all evaluation maps. The representation (1.1) implies that (see e.g., [12,50])

$$\begin{aligned} -\ln \mathbb{P}\{\zeta_Z(t_i) \le x_i, 1 \le i \le n\} &= \mathbb{E}\left\{\max_{1\le i\le n} e^{Z(t_i)-x_i}\right\} = \mathbb{E}\left\{e^{Z(h)} \max_{1\le i\le n} e^{Z(t_i)-Z(h)-x_i}\right\} \\ &= \mathbb{E}\left\{\max_{1\le i\le n} e^{\Xi_h Z(t_i)-x_i}\right\} \end{aligned} \tag{1.4}$$

holds for $t_i \in \mathcal{T}, x_i \in \mathbb{R}, i \le n$ i.e.,

$$\zeta_Z \stackrel{fdd}{=} \zeta_{\Xi_h Z}. \tag{1.5}$$

Since as mentioned above the process $\Xi_h Z$ can be characterised without making reference to σ^2, by (1.5) it follows that the law of ζ_Z depends on γ only!

Observe that we can define $Z^{[h]}$ via exponential tilting for any random process Z such that $\mathbb{E}\{e^{Z(h)}\} = 1$. Furthermore, the calculation of the fidi's of ζ_Z via (1.4) does not refer to the

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