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EXTREMES OF q-ORNSTEIN–UHLENBECK PROCESSES

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Abstract. Two limit theorems are established on the extremes of a family of stationary Markov processes, known as q-Ornstein–Uhlenbeck processes with $q \in (-1, 1)$. Both results are crucially based on the weak convergence of the tangent process at the lower boundary of the domain of the process, a positive self-similar Markov process little investigated so far in the literature. The first result is the asymptotic excursion probability established by the double-sum method, with an explicit formula for the Pickands constant in this context. The second result is a Brown–Resnick-type limit theorem on the minimum process of i.i.d. copies of the q-Ornstein–Uhlenbeck process: with appropriate scalings in both time and magnitude, a new semi-min-stable process arises in the limit.

1. Introduction

In this paper, we continue our investigation on the path properties of q-Ornstein–Uhlenbeck processes ($q \in (-1, 1)$) in [11, 48]. These are stationary Markov processes with explicit transition probability density functions. This family of processes have two origins. On one hand, as Markov processes they, more precisely certain transformations of them called q-Brownian motions, arise as a special case of the quadratic harnesses introduced in [10, 12]. In short, quadratic harnesses are centered and square-integrable stochastic processes $\{X_t\}_{t\in[0,\infty)}$ such that $\mathbb{E}(X_sX_t) = \min(s, t)$ and for $0 \le r < s < t$ given the past $\{X_u\}_{u\le r}$ and future $\{X_u\}_{u\ge t}$, the conditional mean and variances of X_s are in linear and quadratic forms of (X_r, X_t), respectively. On the other hand, the q-Ornstein–Uhlenbeck processes and q-Brownian motions arise for the first time in non-commutative probability with the same name, and it is known since the seminal results of Biane [3] and Bożejko et al. [6] that every non-commutative Markov process has a classical Markov process counterpart that we investigate here. In this paper, we shall focus on q-Ornstein–Uhlenbeck processes as classical Markov processes. No knowledge of non-commutative probability is needed.

As the name tells, the q-Ornstein–Uhlenbeck process has an intriguing connection to the well investigated Ornstein–Uhlenbeck Gaussian process: as $q \uparrow 1$, the former converges weakly to the latter. This makes one wonder to what extend the two processes are similar. For each q fixed, however, we have seen that in terms of path properties, the q-Ornstein–Uhlenbeck processes are qualitatively different [11, 48]. For example, each q-Ornstein–Uhlenbeck process has bounded state space $[-2/\sqrt{1-q}, 2/\sqrt{1-q}]$, while the Ornstein–Uhlenbeck process takes values in $\mathbb{R}$.

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