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Weak order in averaging principle for stochastic wave equations with a fast oscillation

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Abstract

This article deals with the weak error in averaging principle for a stochastic wave equation on a bounded interval $[0, L]$, perturbed by an oscillating term arising as the solution of a stochastic reaction–diffusion equation evolving on the fast time scale. Under suitable conditions, it is proved that the rate of weak convergence of the original solution to the solution of the corresponding averaged equation is of order 1 via an asymptotic expansion approach.

MSC: primary 60H15; secondary 70K70

Keywords: Stochastic wave equation; Averaging principle; Invariant measure; Weak convergence; Asymptotic expansion

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1. Introduction

Let $D = (0, L) \subset \mathbb{R}$ be a bounded open interval. In the article, for any $T > 0$, we consider the following stochastic wave equation with a rapidly oscillating perturbation:

$$\begin{cases} \frac{\partial^2}{\partial t^2} U_t^\epsilon(\xi) = \Delta U_t^\epsilon(\xi) + F(U_t^\epsilon(\xi), Y_t^\epsilon(\xi)) + \sigma_1 \dot{W}_t^1(\xi), \ t \in (0, T], \xi \in D, \\ U_t^\epsilon(\xi) = 0, (\xi, t) \in \partial D \times (0, T], \\ U_0^\epsilon(\xi) = x_1(\xi), \frac{\partial}{\partial t} U_t^\epsilon(\xi)\big|_{t=0} = x_2(\xi), \xi \in D, \end{cases} \tag{1.1}$$

where Δ is the Laplacian operator and $Y_t^\epsilon(\xi)$ is governed by the stochastic reaction–diffusion equation

$$\begin{cases} \frac{\partial}{\partial t} Y_t^\epsilon(\xi) = \frac{1}{\epsilon} \Delta Y_t^\epsilon(\xi) + \frac{1}{\epsilon} g(Y_t^\epsilon(\xi)) + \frac{\sigma_2}{\sqrt{\epsilon}} \dot{W}_t^2(\xi), \ t \in (0, T], \xi \in D, \\ Y_t^\epsilon(\xi) = 0, (\xi, t) \in \partial D \times (0, T], \\ Y_0^\epsilon(\xi) = y(\xi), \ \xi \in D. \end{cases} \tag{1.2}$$

Assumptions on the smoothness of the drifts F and g will be given below. The driving noises are of additive type, where $W_t^1(\xi)$ and $W_t^2(\xi)$ are mutually independent $L^2(D)$-valued Wiener processes on a complete stochastic basis $(\Omega, \mathscr{F}, \mathscr{F}_t, \mathbb{P})$, which will be specified later. The noise strength coefficients σ_1 and σ_2 are positive constants. The parameter ϵ is small and positive, which describes the ratio of time scale between processes $U_t^\epsilon(\xi)$ and $Y_t^\epsilon(\xi)$. With this time scale the variable $U_t^\epsilon(\xi)$ is referred to as the slow component and $Y_t^\epsilon(\xi)$ as the fast component.

Eq. (1.1) is an abstract model for a random vibration of an elastic string with a fast oscillating perturbation. More generally, the nonlinear coupled wave-heat equations with fast and slow time scales may describe the thermoelastic wave propagation in a random medium (Chow [9]), the interactions of fluid motion with other forms of waves (Leung [26], Zhang and Zuazua [39]), temperature-dependent wave phenomena (Leung [25]), magneto-elasticity (Rivera and Racke [28]) and biological problems (Choi and Craig Miller [8], Cardetti and Choi [4], Wu and Chen [33]).

Averaging principle plays an important role in the study of asymptotic behavior for slow–fast dynamical systems. It was first introduced by Bogoliubov [2] for deterministic differential equations. The theory of averaging for stochastic ordinary differential equations may be found in Khasminskii [19], the work of Freidlin and Wentzell [13,14], Veretennikov [29,30] and Kifer [22–24]. Further progress on averaging for stochastic dynamical systems with non-Gaussian noise in finite dimensional space was studied by Xu and co-workers [34–38]. Concerning the infinite dimensional case, it is worth quoting the papers by Cerrai and Freidlin [7], Cerrai [5,6], Bréhier [3], Wang and Roberts [32], Fu et al. [16] and Bao et al. [1].

In Fu et al. [15], the asymptotic limit (as ϵ tends to 0) of system (1.1) was explored within averaging framework. Under suitable conditions, it can be shown that a reduced stochastic wave equation, without the fast component, can be constructed to characterize the essential dynamics of system (1.1) in the pathwise sense, as it was done in Cerrai and Freidlin [7] and Cerrai [5,6] for stochastic partial differential equations of parabolic type and for stochastic ordinary differential equations in Givon et al. [18], Liu [27] and Wainrib [31].

In this article, we are interested in the rate of weak convergence of the slow motion $U_t^\epsilon(\xi)$ to the averaged dynamics. Namely, we will determine the order, with respect to scale parameter ϵ, of weak deviation between the original solution of the slow equation and the solution of