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Large deviations for the empirical measure of a diffusion via weak convergence methods

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Received 14 May 2017; received in revised form 31 August 2017; accepted 25 September 2017
Available online xxxx

Abstract

We consider the large deviation principle for the empirical measure of a diffusion in Euclidean space, which was first established by Donsker and Varadhan. We employ a weak convergence approach and obtain a characterization for the rate function that is dual to the one obtained by Donsker and Varadhan, and which allows us to evaluate the variational form of the rate function for both reversible and nonreversible diffusions.
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MSC: 60F10; 60H10; 60J60
Keywords: Diffusion; Empirical measure; Large deviations; Weak convergence method

1. Introduction

In this work we employ a weak convergence approach to prove the large deviation principle (LDP) for the empirical measure of a diffusion in Euclidean space. Given a positive integer d, let $\mathbb{R}^d$ denote d-dimensional Euclidean space, $\mathcal{B}(\mathbb{R}^d)$ be the σ-algebra of Borel subsets of $\mathbb{R}^d$ and $\mathcal{P}(\mathbb{R}^d)$ be the space of probability measures on $(\mathbb{R}^d, \mathcal{B}(\mathbb{R}^d))$ equipped with the topology of weak convergence. Let $\mathbb{M}^{d\times d}$ denote the set of real-valued $d \times d$ nonnegative definite symmetric matrices. Suppose a and b are continuous functions on $\mathbb{R}^d$ taking values in $\mathbb{M}^{d\times d}$ and $\mathbb{R}^d$, respectively, and for each x in $\mathbb{R}^d$ let $\sigma(x) \in \mathbb{M}^{d\times d}$ be the unique nonnegative definite square

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https://doi.org/10.1016/j.spa.2017.09.020
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Please cite this article in press as: P. Dupuis, D. Lipshutz, Large deviations for the empirical measure of a diffusion via weak convergence methods, Stochastic Processes and their Applications (2017), https://doi.org/10.1016/j.spa.2017.09.020.

root of $a(x)$. Consider the stochastic differential equation (SDE)

$$dX(t) = b(X(t))dt + \sigma(X(t))dW(t), \tag{1.1}$$

where $X = \{X(t), t \geq 0\}$ is a d-dimensional continuous process, $W = \{W(t), t \geq 0\}$ is a d-dimensional Brownian motion and the integral with respect to dW is the Itô integral. Given a solution X of the SDE (1.1), define the associated empirical measure process $L = \{L_t, t > 0\}$ taking values in $\mathcal{P}(\mathbb{R}^d)$, for $t > 0$, by

$$L_t(A) \doteq \frac{1}{t}\int_0^t \delta_{X(s)}(A)ds, \qquad A \in \mathcal{B}(\mathbb{R}^d). \tag{1.2}$$

Here $\delta_x \in \mathcal{P}(\mathbb{R}^d)$ is the Dirac delta measure at $x \in \mathbb{R}^d$. In this work we establish the LDP for the family $\{L_t, t > 0\}$ on $\mathcal{P}(\mathbb{R}^d)$ as $t \to \infty$.

The LDP for the empirical measure process $\{L_t, t > 0\}$ is a classical result that follows from the works of Donsker and Varadhan [5–7], which established the LDP for the empirical measure process associated with a large class of discrete and continuous time Markov processes. Let $(\mathcal{L}, \mathcal{D})$ denote the infinitesimal generator associated with a diffusion X satisfying the SDE (1.1). Then $C_b^2(\mathbb{R}^d)$, the space of twice continuously differentiable real-valued functions on $\mathbb{R}^d$ whose first and second partial derivatives are bounded, lies in $\mathcal{D}$ and for $\phi \in C_b^2(\mathbb{R}^d)$,

$$\mathcal{L}\phi \doteq \frac{1}{2}\sum_{i=1}^d\sum_{j=1}^d a_{ij}\frac{\partial^2\phi}{\partial x_i \partial x_j} + \sum_{i=1}^d b_i \frac{\partial \phi}{\partial x_i}. \tag{1.3}$$

Let $\mathcal{D}^+$ denote the subset of functions $\phi \in \mathcal{D}$ that are uniformly bounded below by a positive constant on $\mathbb{R}^d$. Donsker and Varadhan [7] showed that under certain stability and regularity conditions, $\{L_t, t > 0\}$ satisfies the LDP on $\mathcal{P}(\mathbb{R}^d)$ with rate function $J : \mathcal{P}(\mathbb{R}^d) \to [0, \infty]$ defined, for $\mu \in \mathcal{P}(\mathbb{R}^d)$, by

$$J(\mu) \doteq \sup_{\phi\in\mathcal{D}^+} -\int_{\mathbb{R}^d} \frac{(\mathcal{L}\phi)(x)}{\phi(x)}\mu(dx). \tag{1.4}$$

Other approaches have been developed by Gärtner [12] and Fleming, Sheu and Soner [11] for the empirical measure of Markov processes taking values in compact manifolds; and by Feng and Kurtz [10, Chapter 12] for the empirical measure of Markov processes taking values in more general metric spaces.

We use weak convergence methods to prove the LDP for the empirical measure of a diffusion in Euclidean space. These techniques have been developed more generally for large deviation problems in the book by Dupuis and Ellis [8], and for empirical measures associated with specific classes of continuous time Markov processes in [3,4,9]. There are a couple advantages of our approach that we highlight here. First, we use standard techniques in the theory of weak convergence and SDEs, which are applied directly to the continuous time diffusion process. We find this probabilistic approach to the classical problem appealing. Second, we obtain a variational formulation of the rate function (see (3.3)) that is dual to the one defined in (1.4), and we evaluate the variational form of the rate function for the large class of both reversible and nonreversible diffusions in Euclidean space that are strong solutions of the SDE (1.1) (see Proposition 6.5). In particular, the rate function evaluated at a probability measure μ is the μ-weighted L^2-cost of a feedback control u that is reversible with respect to μ and such that μ is formally invariant under the infinitesimal generator associated with the u-controlled diffusion.

In summary, the main contributions of this work are as follows:

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