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Asymptotic behaviour of high Gaussian minima

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Abstract

We investigate what happens when an entire sample path of a smooth Gaussian process on a compact interval lies above a high level. Specifically, we determine the precise asymptotic probability of such an event, the extent to which the high level is exceeded, the conditional shape of the process above the high level, and the location of the minimum of the process given that the sample path is above a high level.

MSC: primary 60G15; 60F10; secondary 60G70

Keywords: Gaussian process; High excursions; Minima; Precise asymptotics

1. Introduction

Extremal behaviour of Gaussian processes has been the subject of numerous studies. It is of interest from the point of view of the extreme value theory, or large deviations theory and of the theory of sample path properties of stochastic processes. The asymptotic distribution of the supremum of bounded Gaussian processes has been very thoroughly studied; highlights include Dudley [5], Berman and Kôno [4] and Talagrand [9], and the books of Piterbarg [2,7] and Azaïs and Wschebor [3]. In this paper we are interested in another type of the asymptotic behaviour of Gaussian processes: the situation when an entire sample path of the process is above a high level.

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Such situations are important for understanding the structure of the high level excursion sets of Gaussian random processes and fields.

Very loosely speaking, we are interested in the asymptotics of the Gaussian minima when these minima are high. Dealing with high Gaussian minima is not easy. A finite-dimensional situation (in the language of dependent lognormal random variables) is considered in Guliashvili and Tankov [6]. We, on the other hand, consider minima of zero mean sample continuous Gaussian processes. The processes we consider are often stationary, but some nonstationary processes fall within our framework as well.

We now describe the questions of interest to us more concretely. Let $\mathbf{X} := (X_t : t \in \mathbb{R})$ be a centred Gaussian process with continuous paths, defined on some probability space $(\Omega, \mathcal{F}, \mathrm{P})$. Let $[a, b]$ be a compact interval, and let $u > 0$ be a high level. We study a number of problems related to the situation described above, i.e. the situation when the entire sample path $(X_t : t \in [a, b])$ lies above the level u. Specifically, we are interested in the following questions.

Question 1. What is the precise asymptotic behaviour of the probability

$$\mathrm{P}\left(\min_{a\le t\le b} X_t > u\right)$$

as $u \to \infty$?

Question 2. Given the event

$$B_u := \left\{\min_{a\le t\le b} X_t > u\right\}, \tag{1.1}$$

how does the conditional distribution of $(X_t : t \in [a, b])$ behave as $u \to \infty$?

Question 3. Conditionally on B_u, what can be said about the asymptotics of the overshoot

$$\min_{a\le t\le b} X_t - u\,,$$

as $u \to \infty$?

Question 4. Consider the location of the minimum of the process,

$$\arg\min_{a\le t\le b} X_t$$

taken to be the leftmost location of the minimum in case there are ties (it is elementary that this location is a well defined random variable). What is the asymptotic distribution of the location of the minimum given B_u, as $u \to \infty$?

Some information on Questions 1 and 2 is contained in [1]. Regarding Question 1, the latter paper describes the probabilities of the type $\mathrm{P}\left(\min_{a\le t\le b} X_t > u\right)$ on the logarithmic level, while in the present paper we are interested in precise asymptotics of that probability. Regarding Question 2, the latter paper studies the asymptotic behaviour of the ratio

$$\frac{1}{u} X_t,\ a \le t \le b$$

given B_u, as $u \to \infty$, while in the present paper we would like to know the deviations of the sample path from this linear in u behaviour. Furthermore, the paper of Adler et al. [1] provides no information on Questions 3 and 4 above.

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