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Spread of a catalytic branching random walk on a multidimensional lattice☆

Ekaterina Vl. Bulinskaya

Lomonosov Moscow State University, Faculty of Mathematics and Mechanics, Leninskie gory 1, Moscow 119991, Russia

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Abstract

For a supercritical catalytic branching random walk on $\mathbb{Z}^d$, $d \in \mathbb{N}$, with an arbitrary finite catalysts set we study the spread of particles population as time grows to infinity. It is shown that in the result of the proper normalization of the particles positions in the limit there are a.s. no particles outside the closed convex surface in $\mathbb{R}^d$ which we call the propagation front and, under condition of infinite number of visits of the catalysts set, a.s. there exist particles on the propagation front. We also demonstrate that the propagation front is asymptotically densely populated and derive its alternative representation.

MSC 2010: 60J80; 60F15

Keywords: Branching random walk; Supercritical regime; Spread of population; Propagation front; Many-to-one lemma

1. Introduction

Theory of branching processes is a vast and rapidly developing area of probability theory having a multitude of applications (see, e.g., monographs [19] and [22]). A branching process is intended to describe evolution of population of individuals (particles) which could be genes, bacteria, humans, clients waiting in a queue etc. A special section of that theory is constituted by processes in which particles besides producing offspring also move in space. Such a scenario where the motion of a particle is governed by random walk is named a branching random

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E-mail address: bulinskaya@yandex.ru.

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walk (for random walk, see, e.g., books [23] and [5]). One of the most natural and intriguing questions related to branching random walk is how the particles population spreads in the space whenever it survives. Within the last decades a lot of attention has been paid to that question in the framework of different models of branching random walk on integer lattices or in Euclidean space. One can list publications since the paper [3] till numerous recent works, for instance, papers [2,14,24,25] and the monograph [29]. However, those results only slightly concern the model of *catalytic branching random walk* (CBRW) on $\mathbb{Z}^d$, $d \in \mathbb{N}$, with a finite set of catalysts, which is considered here. A specific trait of CBRW is its non-homogeneity in space, i.e. particles may produce offspring only at selected "catalytic" points of $\mathbb{Z}^d$ and the set of these points where catalysts are located is finite. This model is closely related to the so-called parabolic Anderson problem (see, e.g., [18]) and requires special research methods.

Study of different variants of CBRW goes back to more than 10 years (see, e.g., [1] and [30]), although most of papers in this research domain have been published recently, see, for instance, [7,9,16,21,27,31] and [13]. A lot of them analyze asymptotic behavior of total and local particles numbers as time tends to infinity and only few investigate the spread of CBRW. Analysis of the mean total and local particles numbers implemented in the most general form in [10] as well as the strong and weak limit theorems established in [11] shows that CBRW can be classified as supercritical, critical and subcritical like ordinary branching processes and only in the supercritical regime the total and local particles numbers grow jointly to infinity. For this reason, it is of primary interest to consider spread of particles population in supercritical CBRW.

The following advances in the study of CBRW spread have been achieved. The paper [13] devoted to CBRW on $\mathbb{Z}$ reveals that the maximum of CBRW (i.e., the rightmost particle location) increases asymptotically linearly in time tending to infinity. Its authors employ the many-to-few lemma proved in general form in [20], martingale technique and renewal theorems. A similar assertion for catalytic branching Brownian motion on $\mathbb{R}$ with binary fission and a single catalyst is established in [4] among other results. S. Molchanov and E. Yarovaya in their papers such as [27] study the spread of CBRW with binary fission and symmetric random walk on $\mathbb{Z}^d$ by employing the operator theory methods for symmetric evolution operator. Note that in [15] the authors apply the continuous-space counterpart of such CBRW to modeling of homopolymers.

The main aim of our paper is to study the spread of CBRW on $\mathbb{Z}^d$ for arbitrary positive integer d. In contrast to the one-dimensional case where the maximum of CBRW on $\mathbb{Z}$ was investigated, one cannot directly extend the same approach to multidimensional lattices and employ the fundamental martingale techniques as in [13]. The point is that the concept of maximum is indefinite for CBRW on $\mathbb{Z}^d$, $d > 1$. if the random walk is symmetric and catalysts are positioned symmetrically, as well as the starting point of CBRW be at the origin, then it would be sufficient to consider the maximum of the norm of particle locations or the maximal displacement of a particle, similar to [25]. However, in a more general setting it is of interest to understand not only how far a particle can move from the origin but also in which direction such displacement takes place. So, in this paper, we introduce the concept of the propagation front $\mathcal{P} \subset \mathbb{R}^d$ of the particles population as follows. Divide by t the position coordinates of each particle existing in CBRW at time t and let t tend to infinity. Then in the limit there are a.s. no particles outside the set bounded by the closed surface $\mathcal{P}$ and, under condition of infinite number of visits of catalysts, a.s. there exist particles on $\mathcal{P}$. Thus, under this condition, non-random set $\mathcal{P}$ asymptotically separates a.s. population areal and its a.s. void environment. Moreover, we establish that each point of $\mathcal{P}$ is a limiting point for the normalized particles positions in CBRW and derive an alternative representation for the propagation front $\mathcal{P}$. The latter formula allows us to evaluate directly (without any computer simulation) the set $\mathcal{P}$ for a number of examples

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