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Weak atomic convergence of finite voter models toward Fleming–Viot processes

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Abstract

We consider the empirical measures of multi-type voter models with mutation on large finite sets, and prove their weak atomic convergence in the sense of Ethier and Kurtz (1994) toward a Fleming–Viot process. Convergence in the weak atomic topology is strong enough to answer a line of inquiry raised by Aldous (2013) concerning the distributions of the corresponding entropy processes and diversity processes for types.
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1. Introduction

Voter models are a generalization of Moran processes [27] from population genetics in the presence of spatial structure, and have been one of the major subjects in interacting particle systems [26] along with their variants in models of cancer, social dynamics, and probabilistic evolutionary games. See, for example, [2,7–9,13], and the references there for origins of these models. The present paper is a continuation of Chen, Choi and Cox [10] which considers the weak convergence of two-type voter models toward the Wright–Fisher diffusion. Our main goal

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here is to show that with respect to the weak atomic convergence introduced by Ethier and Kurtz [20], which is finer than the usual weak convergence, infinite-type voter models on large finite sets in the presence of mutation converge to a Fleming–Viot process. Fleming–Viot processes are one of the most well studied classes of measure-valued processes, in part due to its duality with the Kingman coalescent (see [6,15,16,19] and many others).

For the voter models considered throughout this paper we allow multiple types and mutation. The models are defined as follows. With respect to a finite set E with size $\#E = N \geq 2$ and a compact metric type space S, a multi-type voter model is defined by a "voting mechanism" according to an irreducible transition kernel q on E with zero trace $q(x,x) \equiv 0$, and incorporates independent mutation according to a finite measure μ on S. The kernel q plays the role of a "voting kernel" in that at rate 1, the type at each $x \in E$ is replaced by the type at y with probability $q(x,y)$. On the other hand, if μ is nonzero, mutation at each $x \in E$ occurs independently with rate $\mu(\mathbb{1})$, and a new type is chosen according to $\mu(\,\cdot\,)/\mu(\mathbb{1})$. The canonical examples for these voter models are defined by voting kernels which are random walk transition probabilities on finite, connected, edge-weighted graphs without self-loops. Here and in what follows, see [4] for terminology in graph theory.

Voter models where voting kernels are random walk transition probabilities on complete graphs reduce to the classical Moran processes. This particular case forms the basis of several important probability models. For example, in the limit of large N, frequencies of a fixed type under two-type Moran processes converge to the Wright–Fisher diffusion. Furthermore, in the multi-type case, the empirical measures of the corresponding Moran processes converge to a Fleming–Viot process, which is a measure-valued infinite dimensional generalization of the Wright–Fisher diffusion (cf. [17, Chapter 10], and also Section 3 below). Along these lines, one of the major interests has been in the rich properties of the continuum limits, whereas the mean-field nature of Moran processes allows complete characterizations of those dynamical equations on their own.

The objects of this paper are the empirical measures of voter models on large finite spatial structures. The empirical measure associated with an (E,q,μ)-voter model (ξ_t) is given by the process $\big(\mathbf{m}(\xi_t)\big)$ taking values in the space $\mathscr{P}(S)$ of probability measures on S. Here, the probability-measure-valued functional $\mathbf{m}$ is defined by

$$\mathbf{m}(\xi) \doteq \sum_{x\in E} \pi(x)\delta_{\xi(x)}, \quad \xi \in S^E, \tag{1.1}$$

where the weight function π is the unique stationary distribution associated with the voting kernel q. Notice that the mass of $\mathbf{m}(\xi)$ at σ, for $\sigma \in S$, gives the (π-weighted) density of type σ in ξ.

The first main result of this paper, Theorem 4.1 below, generalizes the classical result of convergence of multi-type Moran processes to a Fleming–Viot process, and is an infinite-dimensional generalization of [10, Theorem 2.2] where two-type voter models without mutation are investigated. With respect to an appropriate sequence of $(E_n, q^{(n)}, \mu_n)$-voter models $(\xi_t^{(n)})$ where $\#E_n$ increases to infinity, we establish the weak convergence

$$\mathbf{m}\big(\xi^{(n)}_{\gamma_n \cdot}\big) \xrightarrow[n\longrightarrow\infty]{} X \tag{1.2}$$

as probability-measure-valued processes on the type space S (with respect to Skorokhod's J_1-topology), where X is a Fleming–Viot process. Here, the time scales γ_n are growing constants given by the expected first meeting times of two independent Markov chains which are driven by the corresponding voting kernels and are started from stationarity, and $\mathscr{P}(S)$ is equipped