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Absolute continuity of the invariant measure in piecewise deterministic Markov Processes having degenerate jumps

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Abstract

We consider piecewise deterministic Markov processes with degenerate transition kernels of the *house-of-cards-* type. We use a splitting scheme based on jump times to prove the absolute continuity, as well as some regularity, of the invariant measure of the process. Finally, we obtain finer results on the regularity of the one-dimensional marginals of the invariant measure, using integration by parts with respect to the jump times.

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Keywords: Piecewise deterministic Markov processes; Invariant measure; Integration by parts based on jump times; Jump processes

1. Introduction

We consider an interacting particle system $X_t = (X_t^1, \ldots, X_t^N)$ taking values in $\mathbb{R}^N$ and solving, for $t \geq 0$,

$$X_t = X_0 + \int_0^t b(X_s)ds + \sum_{i=1}^N \int_0^t \int_0^\infty a_i(X_{s-})1_{\{z \leq f_i(X_{s-})\}} N^i(ds, dz). \tag{1.1}$$

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Here, $N^i(ds, dz)$, $1 \leq i \leq N$, are independent Poisson random measures on $\mathbb{R}_+ \times \mathbb{R}_+$ having intensity measure $dsdz$ each. The function $b : \mathbb{R}^N \to \mathbb{R}^N$ is a smooth drift function, and for each $1 \leq i \leq N$, $a_i : \mathbb{R}^N \to \mathbb{R}^N$ are jump functions, and $f_i : \mathbb{R}^N \to \mathbb{R}_+$ jump rate functions. The infinitesimal generator of the process X is given for any smooth test function $g : \mathbb{R}^N \to \mathbb{R}$ by

$$Lg(x) = \sum_{i=1}^{N} f_i(x) \left[g(x + a_i(x)) - g(x) \right] + \langle \nabla g(x), b(x) \rangle. \tag{1.2}$$

We work under the assumption that there exists a unique non-exploding solution to (1.1) which is recurrent in the sense of Harris having a unique invariant probability measure m.

The aim of the present paper is to study the smoothness of the invariant measure m of this particle system in the case of degenerate transitions which are of the form

$$x + a_i(x) = \begin{pmatrix} x^1 + a_i^1(x) \\ \vdots \\ x^{i-1} + a_i^{i-1}(x) \\ 0 \\ x^{i+1} + a_i^{i+1}(x) \\ \vdots \\ x^N + a_i^N(x) \end{pmatrix} \leftarrow \text{ coordinate } i, \tag{1.3}$$

for $x = (x^1, \ldots, x^N) \in \mathbb{R}^N$. This means that a jump of particle i leads to a reset of this particle's position to 0 – we call this jump a main jump – and gives an additional $a_i^j(x)$ to any other particle j—we call these jumps collateral jumps following the terminology proposed in [1]. We call such processes *house-of-cards-* like interacting particle systems. Systems of this type are good models for systems of interacting neurons as introduced by Galves and Löcherbach (2016) [19], see also Duarte and Ost (2016) [15] and Hodara et al. (2016) [20].

Notice that (1.1) is a Piecewise Deterministic Markov process (PDMP) in the sense of Davis (1993) [13]. The process evolves according to the deterministic flow $\gamma_{s,t}(x)$ solution of

$$\gamma_{s,t}(x) = x + \int_s^t b(\gamma_{s,u}(x))du, \; s \leq t,$$

between successive jumps, and the only randomness is given by the random jump times and the choice of the (random) positions of the process right after the jump. The jump rate of the process depends on the configuration of the process and is given by $\bar{f}(x) = \sum_{i=1}^N f^i(x)$. The transition kernel $Q(x, dy) = \sum_{i=1}^N \frac{f_i(x)}{\bar{f}(x)} \delta_{x+a_i(x)}(dy)$ is (partly) degenerate if the jumps are governed by transitions as described in (1.3); indeed, in this case, not only transitions do not create density, but they even destroy density for the particles that jump — which are reset to 0. As a consequence, we are in a very singular scheme here.

Invariant measures and densities of PDMP's or more generally of jump processes have been widely studied in the literature. An overwhelming number of articles is devoted to the study of the regularity of the transition semi-group, i.e. the study of the existence and regularity of a transition density. For this purpose, the Malliavin calculus for processes with jumps has been developed, using the regularity both created by the jump amplitudes or the jump times. We refer to the by now classical studies of Bichteler, Gravereaux and Jacod (1987) [7], Bismut (1983) [9], Carlen and Pardoux (1990) [10], Denis (2000) [14] and Picard (1996) [23]. These papers deal with a

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