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Stochastic porous media equation on general measure spaces with increasing Lipschitz nonlinearities ☆

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Abstract

We prove the existence and uniqueness of probabilistically strong solutions to stochastic porous media equations driven by time-dependent multiplicative noise on a general measure space $(E, \mathscr{B}(E), \mu)$, and the Laplacian replaced by a negative definite self-adjoint operator L. In the case of Lipschitz nonlinearities Ψ, we in particular generalize previous results for open $E \subset \mathbb{R}^d$ and L=Laplacian to fractional Laplacians. We also generalize known results on general measure spaces, where we succeeded in dropping the transience assumption on L, in extending the set of allowed initial data and in avoiding the restriction to superlinear behavior of Ψ at infinity for $L^2(\mu)$-initial data.

Keywords: Wiener process; Porous media equation; Sub-Markovian contractive semigroup

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1. Introduction

In this paper, we consider stochastic porous media equations (SPMEs) of the following type:

$$\begin{cases} dX(t) - L\Psi(X(t))dt = B(t, X(t))dW(t), \text{ in } [0,T]\times E, \\ X(0) = x \text{ on } E \text{ (with } x \in F_{1,2}^* \text{ or } L^2(\mu)), \end{cases} \tag{1.1}$$

where L is the negative definite self-adjoint generator of a sub-Markovian strongly continuous contraction semigroup $(P_t)_{t\geq 0}$ on $L^2(\mu) := L^2(E, \mathscr{B}(E), \mu)$, and $(E, \mathscr{B}(E), \mu)$ is a σ-finite measure space. $\Psi(\cdot) : \mathbb{R} \to \mathbb{R}$ is a monotonically nondecreasing Lipschitz continuous function, B is a progressively measurable process in the space of Hilbert–Schmidt operator from $L^2(\mu)$ to $F_{1,2}^*$, $W(t)$ is an $L^2(\mu)$-valued cylindrical $\mathscr{F}_t$-adapted Wiener process on a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ with normal filtration $(\mathscr{F}_t)_{t\geq 0}$. For the definition of the Hilbert space $F_{1,2}^*$ and the precise conditions on B we refer to the next section.

In the special case when $E = \mathbb{R}^d$, L is equal to the Laplace operator Δ and B is time-independent linear multiplicative, Eq. (1.1) was recently analyzed in [2]. The aim of this paper is to prove analogous results as in [2] for the general case. The above framework is inspired by the work of Fukushima and Kaneko [3] (see also [5]).

The main motivation for this generality is that we would like to cover fractional powers of the Laplacian, i.e., $L = -(-\Delta)^\alpha$, $\alpha \in (0,1)$, generalized Schrödinger operators, i.e., $L = \Delta + 2\frac{\nabla\rho}{\rho}\cdot\nabla$, and Laplacians on fractals (see Section 4).

Recently, there has been much work on stochastic versions of the porous media equations. Based on the variational approach and monotonicity assumptions on the coefficients, [13] presents a generalization of Krylov–Rozovskii's result [8] on the existence and uniqueness of solutions to monotone stochastic differential equations, which applies to a large class of stochastic porous media equations. It should be said that in [13] (see also [14]), Ψ is assumed to be continuous such that $r\Psi(r) \to \infty$ as $r \to \infty$. In this paper we show that for Lipschitz continuous Ψ this condition can be dropped for initial data in $L^2(\mu)$, extending the corresponding result from [2] to general operators L as above. We would also like to emphasize that in contrast to [13,14], in this paper, we do not assume that L is the generator of a transient Dirichlet form on $L^2(E, \mathscr{B}(E), \mu)$. In our case we can drop the transience assumption. In particular, in contrast to [13] (and [14]), we do not need any restriction on d when $E = \mathbb{R}^d$ and $L = -(-\Delta)^\alpha$, $\alpha \in (0,1]$. For more references on stochastic porous media equations we refer to [1]. In addition, we work in the state space $F_{1,2}^*$ which is larger than the state space $\mathscr{F}_e^*$ considered in [13], hence we can allow more general initial conditions (as done in [14] under assumptions much stronger than transience).

Section 4 of [2] deals with the case where Ψ is a maximal monotone multivalued function with at most polynomial growth. However, due to the multiplier problem, the existence is obtained for $d \geq 3$ only. We plan to extend also this result to our more general equation (1.1). This will be the subject of our future work.

The paper is organized as follows: in Section 2, we recall some notions concerning sub-Markovian semi-groups and introduce a suitable Gelfand triple. Section 3 is devoted to verify the existence and uniqueness of strong solutions to (1.1). Note that the Riesz isomorphism $1 - L$, through which we identify $H := F_{1,2}^*$ and $H^* := F_{1,2}$, plays an essential role in the proof. In Section 4, we will apply our results to a number of examples.

2. Preliminaries

First of all, let us recall some basic definitions and spaces which will be used throughout the paper (see [3–5]).

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