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Favorite sites of randomly biased walks on a supercritical Galton–Watson tree[☆]

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Abstract

Erdős and Révész (1984) initiated the study of favorite sites by considering the one-dimensional simple random walk. We investigate in this paper the same problem for a class of null-recurrent randomly biased walks on a supercritical Galton–Watson tree. We prove that there is some parameter $\kappa \in (1, \infty]$ such that the set of the favorite sites of the biased walk is almost surely bounded in the case $\kappa \in (2, \infty]$, tight in the case $\kappa = 2$, and oscillates between a neighborhood of the root and the boundary of the range in the case $\kappa \in (1, 2)$. Moreover, our results yield a complete answer to the cardinality of the set of favorite sites in the case $\kappa \in (2, \infty]$. The proof relies on the exploration of the Markov property of the local times process with respect to the space variable and on a precise tail estimate on the maximum of local times, using a change of measure for multi-type Galton–Watson trees.

MSC: 60J80; 60G50; 60K37

Keywords: Biased random walk on the Galton–Watson tree; Local times; Favorite sites; Multitype Galton–Watson tree

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1. Introduction

The study of favorite sites goes back to Erdős and Révész [15] where they considered the simple random walk on $\mathbb{Z}$, and conjectured that

(a) the set of favorite sites is tight;

(b) the cardinality of the set of favorite sites is eventually bounded by 2.

We refer to Révész ([36], Chapter 13) for a list of ten open problems on the favorite sites.

Conjecture (b) still remains open and the best result so far was obtained by Tóth [40]. Conjecture (a) was disproved by Bass and Griffin [7] who showed the almost sure transience of the favorite sites for the simple random walk on $\mathbb{Z}$ as well as for the one-dimensional Brownian motion. We note in passing that the exact rate of escape of the favorite sites is still an open problem. Later, the transience of the favorite sites was established by Bass, Eisenbaum and Shi [6], Marcus [32], Eisenbaum and Khoshnevisan [14] for Lévy processes and even for fairly general Markov processes, and by Hu and Shi [21] for Sinai's one-dimensional random walk in random environment. One may wonder whether the favorite sites are always transient for general "non-trivial" null-recurrent Markov processes. This was however disproved by Hu and Shi [23]: the set of the favorite sites is tight for a class of randomly biased walks on trees in the slow-movement regime. The present paper is to address the same question in the sub-diffusive regime. The answer is more complicated and is depending on some parameter $\kappa \in (1, \infty]$. For a class of biased walk on tree, conditioned upon the set of non-extinction of the tree, the set of favorite sites will be almost surely bounded if $\kappa > 2$, tight if $\kappa = 2$, and may move to infinity almost surely if $1 < \kappa < 2$. As a consequence of our results, we can give a complete answer to the cardinality of the set of favorite sites when $\kappa > 2$.

Let us define now the model of the randomly biased walk on trees, a model introduced by Lyons and Pemantle [31]. Let $\mathbb{T}$ be a supercritical Galton–Watson tree, rooted at $\varnothing$. For any vertex $x \in \mathbb{T} \setminus \{\varnothing\}$, let $\overleftarrow{x}$ be its parent. Let $\omega := (\omega(x, \cdot),\ x \in \mathbb{T})$ be a sequence of vectors such that for each vertex $x \in \mathbb{T}$, $\omega(x,\ y) \geq 0$ for all $y \in \mathbb{T}$ and $\sum_{y\in\mathbb{T}}\omega(x,\ y) = 1$. We assume that $\omega(x,\ y) > 0$ if and only if either $\overleftarrow{x} = y$ or $\overleftarrow{y} = x$. Denote by $|x|$ the generation of the vertex x in $\mathbb{T}$. We shall also use the partial order on the tree: for any $x, y \in \mathbb{T}$, we write $y < x$ if and only if y is an ancestor of x (and $y \leq x$ iff $y < x$ or $y = x$).

For the sake of presentation, we add a specific vertex $\overleftarrow{\varnothing}$, considered as the parent of $\varnothing$. We stress that $\overleftarrow{\varnothing}$ is not a vertex of $\mathbb{T}$, for instance, $\sum_{x\in\mathbb{T}} f(x)$ does not contain the term $f(\overleftarrow{\varnothing})$. We define $\omega(\overleftarrow{\varnothing}, \varnothing) := 1$ and modify the vector $\omega(\varnothing, \cdot)$ such that $\omega(\varnothing, \overleftarrow{\varnothing}) > 0$ and $\omega(\varnothing, \overleftarrow{\varnothing}) + \sum_{x:\,\overleftarrow{x}=\varnothing}\omega(\varnothing, x) = 1$.

For given ω, the randomly biased walk $(X_n)_{n\geq 0}$ is a Markov chain on $\mathbb{T} \cup \{\overleftarrow{\varnothing}\}$ with transition probabilities ω, starting from $\varnothing$; i.e. $X_0 = \varnothing$ and

$$P_\omega\big(X_{n+1} = y \,|\, X_n = x\big) = \omega(x,\ y).$$

For any vertex $x \in \mathbb{T}$, let $(x^{(1)}, \ldots, x^{(\nu_x)})$ be its children, where $\nu_x \geq 0$ is the number of children of x. Define $\mathbf{A}(x) := (A(x^{(i)}),\ 1 \leq i \leq \nu_x)$ by

$$A(x^{(i)}) := \frac{\omega(x,\ x^{(i)})}{\omega(x,\ \overleftarrow{x})}, \qquad 1 \leq i \leq \nu_x\,.$$

We denote the vector $\mathbf{A}(\varnothing)$ by $(A_1, \ldots, A_\nu)$. As such, $\nu \equiv \nu_\varnothing$ is the number of children of $\varnothing$. When ν is a given integer (i.e. $\mathbb{T}$ is a regular tree), we suppose that $(\mathbf{A}(x))_{x\in\mathbb{T}}$ are i.i.d. In general, when ν is also random, we may construct a marked tree as in Neveu [34] such that for any $k \geq 0$,

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