Accepted Manuscript

Covariance of stochastic integrals with respect to fractional Brownian motion

Yohaï Maayan, Eddy Mayer-Wolf

PII: S0304-4149(17)30197-7
DOI: http://dx.doi.org/10.1016/j.spa.2017.08.006
Reference: SPA 3176

To appear in: *Stochastic Processes and their Applications*

Received date : 9 January 2017
Revised date : 12 July 2017
Accepted date : 8 August 2017

Covariance of stochastic integrals with respect to fractional Brownian motion

Yohaï Maayan*, Eddy Mayer-Wolf

Department of Mathematics, Technion, Israel Institute of Technology, 32000 Haifa, Israel

Abstract

We find an explicit expression for the cross-covariance between stochastic integral processes with respect to a d-dimensional fractional Brownian motion (fBm) B_t with Hurst parameter $H > 1/2$, where the integrands are vector fields applied to B_t. It provides, for example, a direct alternative proof of Y. Hu and D. Nualart's result that the stochastic integral component in the fractional Bessel process decomposition is not itself a fractional Brownian motion.

Keywords: fractional Brownian motion, divergence integral, stochastic integral, fractional Bessel process
2010 MSC: 60G15, 60G18, 60G22, 60H05, 60H07

1. Introduction

Fractional Brownian motion is a family of zero mean stationary Gaussian processes $B_t = B_t^H$ indexed by $H \in (0, 1)$ which was mathematically introduced by B.B Mandelbrot and J.W. Van Ness in [1] (cf. [2] as well). It generalizes Brownian motion ($H = \frac{1}{2}$) in that $EB_t^2 = t^{2H}$, and can be used to model various phenomena, in finance as well as in other fields. This is primarily due to the fact that its self-similarity depends on the parameter H,

which allows for phenomena exhibiting different kinds of self-similarity to be modeled by fractional Brownian motion with an appropriate H.

Since fractional Brownian motion is not a semimartingale (unless $H = \frac{1}{2}$), the ordinary stochastic calculus for semimartingales (such as the Itô integral) does not apply. Instead, there are several approaches for defining a stochastic integral with respect to fractional Brownian motion. The divergence integral is one possible approach, the one discussed in this paper, using the Malliavin divergence operator as the basis for integration, a survey of which can be found in D. Nualart's book [3]. One other approach for example was developed by Zähle in [4], which involves a pathwise definition of the stochastic integral. This

*Corresponding author.
Email addresses: ymaayan@tx.technion.ac.il (Yohaï Maayan), emw@tx.technion.ac.il (Eddy Mayer-Wolf)