SPA: 3177 Model 1 pp. 1–26 (col. fig: NIL)

ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Stochastic Processes and their Applications xx (xxxx) xxx–xxx

stochastic processes and their applications

www.elsevier.com/locate/spa

Convex integral functionals of regular processes

Teemu Pennanen[a], Ari-Pekka Perkkiö[b,*]

[a] *Department of Mathematics, King's College London, Strand, London, WC2R 2LS, United Kingdom*
[b] *Mathematics Institute, Ludwig-Maximilian University of Munich, Theresienstr. 39, 80333 Munich, Germany*

Received 26 April 2016; received in revised form 16 June 2017; accepted 9 August 2017
Available online xxxx

Abstract

This article gives dual representations for convex integral functionals on the linear space of regular processes. This space turns out to be a Banach space containing many more familiar classes of stochastic processes and its dual can be identified with the space of optional random measures with essentially bounded variation. Combined with classical Banach space techniques, our results allow for a systematic treatment of stochastic optimization problems over BV processes and, in particular, yields a maximum principle for a general class of singular stochastic control problems.

Keywords: Regular process; Integral functional; Conjugate duality; Singular stochastic control

1. Introduction

This article studies convex integral functionals of the form

$$EI_h(v) = E\int_0^T h_t(v_t)d\mu_t$$

defined on the linear space $\mathcal{R}^1$ of regular processes in a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P)$. Here μ is a positive optional measure on $[0, T]$ and h is a convex normal integrand on $\Omega \times [0, T] \times \mathbb{R}^d$. An optional cadlag process v of class (D) is *regular* if $Ev_{\tau^\nu} \to Ev_\tau$ for every increasing sequence of stopping times τ^ν converging to a finite stopping time τ or equivalently (see [9, Remark 50d]), if the predictable projection and the left limit of v coincide.

* Corresponding author.
E-mail address: perkkioe@math.tu-berlin.de (A.-P. Perkkiö).

http://dx.doi.org/10.1016/j.spa.2017.08.007

Please cite this article in press as: T. Pennanen, A.-P. Perkkiö, Convex integral functionals of regular processes, Stochastic Processes and their Applications (2017), http://dx.doi.org/10.1016/j.spa.2017.08.007.

Regular processes is quite a large family of stochastic processes containing e.g. continuous adapted processes, Levy processes and Feller processes as long as they are of class (D). A semimartingale is regular if and only if it is of class (D) and the predictable BV part of its Doob–Meyer decomposition is continuous.

Inspection of [3] reveals that $\mathcal{R}^1$ is a Banach space under a suitable norm and its dual may be identified with the space $\mathcal{M}^\infty$ of optional random measures with essentially bounded variation. Our main result characterizes the corresponding conjugate and subdifferential of EI_h under suitable conditions on the integrand h. Our main result applies, more generally, to functionals of the form $EI_h + \delta_{\mathcal{R}^1(D)}$, where $\mathcal{R}^1(D)$ denotes the convex set of regular processes that, outside an evanescent set, take values in $D_t(\omega) := \operatorname{cl}\operatorname{dom} h_t(\cdot, \omega)$. Here, as usual, $\delta_{\mathcal{R}^1(D)}$ is the *indicator function* of $\mathcal{R}^1(D)$ taking the value 0 on $\mathcal{R}^1(D)$ and $+\infty$ outside of $\mathcal{R}^1(D)$.

Our main result allows for functional analytic treatment of various stochastic optimization problems where one minimizes an integral functional over the space of BV-processes. Our original motivation came from mathematical finance where BV-processes arise naturally as trading strategies in the presence of transaction costs. In this paper, we give an application to singular stochastic control by deriving a dual problem and a maximum principle for a fairly general class of singular control problems that extends and unifies singular control models of e.g. [2,4,11,18]. Applications to mathematical finance will be given in a separate article.

The main result of this paper combines convex analysis with the general theory of stochastic processes. More precisely, we employ the duality theory of integral functionals on the space of continuous functions developed by [28] combined with Bismut's characterization of regular processes as optional projections of continuous stochastic processes; see [3]. Our main result states that if the conjugate h^* of h is the optional projection of a convex normal integrand that allows for Rockafellar's dual representation of I_h scenario wise, then under mild integrability conditions, the dual representation of $EI_h + \delta_{\mathcal{R}^1(D)}$ is given simply as the expectation of that of $I_h + \delta_{C(D)}$, where $C(D)$ denotes the continuous selections of D. The proof is more involved than the classical results on integral functionals on decomposable spaces or on spaces of continuous functions. To treat the space of regular processes, techniques from both cases need to be combined in a nontrivial way. Our proof is based on recent results on optional projections of normal integrands from [17] and conjugate results for continuous functions from [22].

2. Integral functionals and duality

This section collects some basic facts about integral functionals defined on the product of a measurable space $(\Xi, \mathcal{A})$ and a Suslin locally convex vector space U. In the applications below, Ξ is either Ω, $[0, T]$ or $\Omega \times [0, T]$. Recall that a Hausdorff topological space is *Suslin* if it is a continuous image of a complete separable metric space. We will also assume that U is a countable union of Borel sets that are Polish spaces in their relative topology. Examples of such spaces include separable Banach spaces as well as their topological duals when equipped with the weak*-topology. Indeed, such dual spaces are Suslin [32, Proposition A.9] and their closed unit balls are metrizable in the weak*-topology by [10, Theorem V.5.1], compact by the Banach–Alaoglu theorem, and thus separable by [10, Theorem I.6.25].

A set-valued mapping $S : \Xi \rightrightarrows U$ is *measurable* if the inverse image $S^{-1}(O) := \{\xi \in \Xi \mid S(\xi) \cap O \neq \emptyset\}$ of every open $O \subseteq U$ is in $\mathcal{A}$. An extended real-valued function $f : U \times \Xi \to \overline{\mathbb{R}}$ is said to be a *normal integrand* if the *epigraphical mapping*

$$\xi \mapsto \operatorname{epi} f(\cdot, \xi) := \{(u, \alpha) \in U \times \mathbb{R} \mid f(u, \xi) \leq \alpha\}$$