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Asymptotic properties of maximum likelihood estimator for the growth rate for a jump-type CIR process based on continuous time observations

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Abstract

We consider a jump-type Cox–Ingersoll–Ross (CIR) process driven by a standard Wiener process and a subordinator, and we study asymptotic properties of the maximum likelihood estimator (MLE) for its growth rate. We distinguish three cases: subcritical, critical and supercritical. In the subcritical case we prove weak consistency and asymptotic normality, and, under an additional moment assumption, strong consistency as well. In the supercritical case, we prove strong consistency and mixed normal (but non-normal) asymptotic behavior, while in the critical case, weak consistency and non-standard asymptotic behavior are described. We specialize our results to so-called basic affine jump–diffusions as well. Concerning the asymptotic behavior of the MLE in the supercritical case, we derive a stochastic representation of the limiting mixed normal distribution, where the almost sure limit of an appropriately scaled jump-type supercritical CIR process comes into play. This is a new phenomenon, compared to the critical case, where a diffusion-type critical CIR process plays a role.

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1. Introduction

Continuous state and continuous time branching processes with immigration, especially, the Cox–Ingersoll–Ross (CIR) process (introduced by Feller [16] and Cox et al. [11]) and its variants, play an important role in stochastics, and there is a wide range of applications of these processes in biology and financial mathematics as well. In the framework of the famous Heston model, which is popular in finance, a CIR process can be interpreted as a stochastic volatility (or instantaneous variance) of a price process of an asset. In this paper, we consider a jump-type CIR process driven by a standard Wiener process and a subordinator

$$\mathrm{d}Y_t = (a - bY_t)\,\mathrm{d}t + \sigma\sqrt{Y_t}\,\mathrm{d}W_t + \mathrm{d}J_t, \qquad t \in [0,\infty), \tag{1.1}$$

with an almost surely non-negative initial value Y_0, where $a \in [0,\infty)$, $b \in \mathbb{R}$, $\sigma \in (0,\infty)$, $(W_t)_{t\in[0,\infty)}$ is a 1-dimensional standard Wiener process, and $(J_t)_{t\in[0,\infty)}$ is a subordinator (an increasing Lévy process) with zero drift and with Lévy measure m concentrating on $(0,\infty)$ such that

$$\int_0^\infty z\, m(\mathrm{d}z) \in [0,\infty), \tag{1.2}$$

that is,

$$\mathbb{E}(\mathrm{e}^{uJ_t}) = \exp\left\{ t\int_0^\infty (\mathrm{e}^{uz} - 1)\, m(\mathrm{d}z)\right\} \tag{1.3}$$

for any $t \in [0,\infty)$ and for any complex number u with $\mathrm{Re}(u) \in (-\infty, 0]$, see, e.g., Sato [44, proof of Theorem 24.11]. We suppose that Y_0, $(W_t)_{t\in[0,\infty)}$ and $(J_t)_{t\in[0,\infty)}$ are independent. Note that the moment condition (1.2) implies that m is a Lévy measure, since $\min(1, z^2) \leqslant z$ for $z \in (0,\infty)$. Moreover, the subordinator J has sample paths of bounded variation on every compact time interval almost surely, see, e.g., Sato [44, Theorem 21.9]. We point out that the assumptions assure that there is a (pathwise) unique strong solution of the SDE (1.1) with $\mathbb{P}(Y_t \in [0,\infty) \text{for all} = 1\ t \in [0,\infty)) = 1$ (see Proposition 2.1). In fact, $(Y_t)_{t\in[0,\infty)}$ is a special continuous state and continuous time branching process with immigration (CBI process), see Proposition 2.1.

In the present paper, we focus on parameter estimation for the jump-type CIR process (1.1) in critical and supercritical cases ($b = 0$ and $b \in (-\infty, 0)$, respectively), which have not been addressed in previous research. We also study the subcritical case ($b \in (0,\infty)$) and we get results extending those of Mai [40, Theorem 4.3.1] in several aspects: we do not suppose the ergodicity of the process Y and we make explicit the expectation of the unique stationary distribution of Y in the limit law in Theorem 5.2. However, we note that some points in Mai's approach [40, Sections 3.3 and 4.3] should be corrected concerning the expressions of the likelihood ratio (Mai [40, formula (3.10)]) and the maximum likelihood estimator (MLE) of $b \in \mathbb{R}$ (Mai [40, formula (4.23)]), see our results in Propositions 4.1 and 4.2, respectively. Supposing that $a \in [0,\infty)$, $\sigma \in (0,\infty)$ and the measure m are known, we study the asymptotic

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