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On the normal approximation for random fields via martingale methods

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Abstract

We prove a central limit theorem for strictly stationary random fields under a sharp projective condition. The assumption was introduced in the setting of random sequences by Maxwell and Woodroofe. Our approach is based on new results for triangular arrays of martingale differences, which have interest in themselves. We provide as applications new results for linear random fields and nonlinear random fields of Volterra-type.

MSC: 60F05, 60G10, 60G48

Keywords: Random field; Central limit theorem; Maxwell-Woodroofe condition; Martingale approximation.

1 Introduction

Martingale methods are very important for establishing limit theorems for sequences of random variables. The theory of martingale approximation, initiated by Gordin (1969), was perfected in many subsequent papers. A random field consists of multi-indexed random variables $(X_u)_{u\in Z^d}$. The main difficulty when analyzing the asymptotic properties of random fields, is the fact that the future and the past do not have a unique interpretation. Nevertheless, it is still natural to try to exploit the richness of the martingale techniques. The main problem consists of the construction of meaningful filtrations. In order to overcome this difficulty mathematicians either used the lexicographic order or introduced the notion of commuting filtration. The lexicographic order appears in early papers, such as in Rosenblatt (1972), who pioneered the field of martingale approximation in the context of random fields. An important result was obtained by Dedecker (1998) who pointed out an interesting projective criteria for random fields, also based on the lexicographic order. The lexicographic order leads to normal approximation under projective conditions with respect to rather large, half-plane indexed sigma algebras. In order to reduce the size of the filtration used in projective conditions, mathematicians introduced the so-called commuting filtrations. The traditional way for constructing commuting filtrations is to