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MARTINGALE PROBLEMS FOR SOME DEGENERATE KOLMOGOROV EQUATIONS

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ABSTRACT. We obtain Calderón-Zygmund estimates for some degenerate equations of Kolmogorov type with inhomogeneous nonlinear coefficients. We then derive the well-posedness of the martingale problem associated with related degenerate operators, and therefore uniqueness in law for the corresponding stochastic differential equations. Some density estimates are established as well.

1. INTRODUCTION

1.1. **Statement of the problem.** Consider the following system of Stochastic Differential Equations (SDEs in short)

$$
\begin{array}{ll}
dX_t^1 = F_1(t, X_t^1, \dots, X_t^n)dt + \sigma(t, X_t^1, \dots, X_t^n)dW_t, & \\
dX_t^2 = F_2(t, X_t^1, \dots, X_t^n)dt, & \\
dX_t^3 = F_3(t, X_t^2, \dots, X_t^n)dt, & t \geq 0, \\
\dots & \\
dX_t^n = F_n(t, X_t^{n-1}, X_t^n)dt, &
\end{array}
\tag{1.1}
$$

$(W_t)_{t\geq 0}$ standing for a d-dimensional Brownian motion, and each $(X_t^i)_{t\geq 0}$, $i \in [\![1, n]\!]$, being $\mathbb{R}^d$-valued as well.

From the applicative viewpoint, systems of type (1.1) appear in many fields. Let us for instance mention for $n = 2$ stochastic Hamiltonian systems (see e.g. Soize [Soi94] for a general overview or Talay [Tal02] and Hérau and Nier [HN04] for convergence to equilibrium). Again for $n = 2$, the above dynamics is used in mathematical finance to price Asian options (see for example [BPV01]). For $n \geq 2$, it appears in heat conduction models (see e.g. Eckmann et al. [EPRB99] and Rey-Bellet and Thomas [RBT00] when the chain of differential equations is forced by two heat baths).

Assume first that the coefficients $(F_i)_{i\in[\![1,n]\!]}$ are Lipschitz continuous in space and that the diffusion matrix $a(t, .) := \sigma\sigma^*(t, .)$ is bounded. If we additionally suppose that $a(t, .)$ and $(D_{x_{i-1}} F_i(t, .))_{i\in[\![2,n]\!]}$ are non-degenerate (weak Hörmander condition) and Hölder continuous in space, with respective Hölder exponents in $(1/2, 1]$ and $(0, 1]$, some multi-scale Gaussian Aronson like estimates have been proved in [DM10] for the density of (1.1) uniformly on the time set $(0, T]$, for fixed $T > 0$ (see Example 2 and Theorem 1.1 of that reference). Those results extend to the case of an arbitrary Hölder exponent in $(0, 1]$ for $a(t, .)$ thanks to uniqueness in law arguments that have been investigated in [Men11] through the well posedness

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