

Accepted Manuscript

Martingale problems for some degenerate Kolmogorov equations

Stéphane Menozzi

PII: S0304-4149(17)30156-4
DOI: <http://dx.doi.org/10.1016/j.spa.2017.06.001>
Reference: SPA 3140

To appear in: *Stochastic Processes and their Applications*

Received date: 21 May 2016
Revised date: 6 February 2017
Accepted date: 7 June 2017

Please cite this article as: S. Menozzi, Martingale problems for some degenerate Kolmogorov equations, *Stochastic Processes and their Applications* (2017), <http://dx.doi.org/10.1016/j.spa.2017.06.001>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



1
2
3
4
5
6
7
8
9
10
11 **MARTINGALE PROBLEMS FOR SOME DEGENERATE**
12 **KOLMOGOROV EQUATIONS**
13

14
15 STÉPHANE MENOZZI
16

17
18 **ABSTRACT.** We obtain Calderón-Zygmund estimates for some degenerate equa-
19 tions of Kolmogorov type with inhomogeneous nonlinear coefficients. We then
20 derive the well-posedness of the martingale problem associated with related
21 degenerate operators, and therefore uniqueness in law for the corresponding
22 stochastic differential equations. Some density estimates are established as
23 well.
24

25
26 1. INTRODUCTION
27

28 **1.1. Statement of the problem.** Consider the following system of Stochastic
29 Differential Equations (SDEs in short)

$$\begin{aligned} (1.1) \quad & dX_t^1 = F_1(t, X_t^1, \dots, X_t^n)dt + \sigma(t, X_t^1, \dots, X_t^n)dW_t, \\ & dX_t^2 = F_2(t, X_t^1, \dots, X_t^n)dt, \\ & dX_t^3 = F_3(t, X_t^2, \dots, X_t^n)dt, \\ & \dots \\ & dX_t^n = F_n(t, X_t^{n-1}, X_t^n)dt, \end{aligned} \quad t \geq 0,$$

30
31
32
33
34
35
36 $(W_t)_{t \geq 0}$ standing for a d -dimensional Brownian motion, and each $(X_t^i)_{t \geq 0}$, $i \in$
37 $\llbracket 1, n \rrbracket$, being \mathbb{R}^d -valued as well.

38 From the applicative viewpoint, systems of type (1.1) appear in many fields.
39 Let us for instance mention for $n = 2$ stochastic Hamiltonian systems (see e.g.
40 Soize [Soi94] for a general overview or Talay [Tal02] and Hérau and Nier [HN04]
41 for convergence to equilibrium). Again for $n = 2$, the above dynamics is used in
42 mathematical finance to price Asian options (see for example [BPV01]). For $n \geq 2$,
43 it appears in heat conduction models (see e.g. Eckmann et al. [EPRB99] and Rey-
44 Bellet and Thomas [RBT00] when the chain of differential equations is forced by
45 two heat baths).

46 Assume first that the coefficients $(F_i)_{i \in \llbracket 1, n \rrbracket}$ are Lipschitz continuous in space
47 and that the diffusion matrix $a(t, \cdot) := \sigma \sigma^*(t, \cdot)$ is bounded. If we additionally
48 suppose that $a(t, \cdot)$ and $(D_{x_{i-1}} F_i(t, \cdot))_{i \in \llbracket 2, n \rrbracket}$ are non-degenerate (weak Hörmander
49 condition) and Hölder continuous in space, with respective Hölder exponents in
50 $(1/2, 1]$ and $(0, 1]$, some multi-scale Gaussian Aronson like estimates have been
51 proved in [DM10] for the density of (1.1) uniformly on the time set $(0, T]$, for fixed
52 $T > 0$ (see Example 2 and Theorem 1.1 of that reference). Those results extend to
53 the case of an arbitrary Hölder exponent in $(0, 1]$ for $a(t, \cdot)$ thanks to uniqueness in
54 law arguments that have been investigated in [Men11] through the well posedness
55
56

57 *Date:* February 6, 2017.

58 *2000 Mathematics Subject Classification.* Primary 60H10, 60G46; Secondary 60H30, 35K65.

59 *Key words and phrases.* Degenerate SDEs, martingale problem, Calderón-Zygmund estimates.
60
61
62
63
64
65

Download English Version:

<https://daneshyari.com/en/article/7550370>

Download Persian Version:

<https://daneshyari.com/article/7550370>

[Daneshyari.com](https://daneshyari.com)