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The free energy of the random walk pinning model

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Abstract

We consider the random walk pinning model. This is a random walk on $\mathbb{Z}^d$ whose law is given as the Gibbs measure $\mu_{N,Y}^{\beta}$, where the polymer measure $\mu_{N,Y}^{\beta}$ is defined by using the collision local time with another simple symmetric random walk Y on $\mathbb{Z}^d$ up to time N. Then, at least two definitions of the phase transitions are known, described in terms of the partition function and the free energy. In this paper, we will show that the two critical points coincide and give an explicit formula for the free energy in terms of a variational representation. Also, we will prove that if β is smaller than the critical point, then X under $\mu_{N,Y}^{\beta}$ satisfies the central limit theorem and the invariance principle P_Y-almost surely.

MSC 2010: 60K35; 82B26; 82B44

Keywords: Random walk pinning model; Free energy; Delocalization; Localization; Central limit theorem; Invariance principle

We denote by $(\Omega, \mathscr{F}, P)$ a probability space. We denote by $P[X]$ the expectation of a random variable X with respect to P. Let $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$, $\mathbb{N} = \{1, 2, 3, \ldots\}$, and $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$. For $x = (x_1, \ldots, x_d) \in \mathbb{R}^d$, $|x|$ stands for the l^1-norm: $|x| = \sum_{i=1}^{d} |x_i|$. For $\mathbf{n} = (n_1, \ldots, n_d) \in \mathbb{N}_0^d$, we set $|\mathbf{n}| = \sum_{i=1}^{d} n_i$.

1. Introduction and main results

1.1. Model

The random walk pinning model (RWPM) was introduced by Birkner and Sun [6]. It is known that the model is related to the parabolic Anderson model with a single moving catalyst, the

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pinning and copolymer model, and the directed polymers in random environment. We refer the readers to [6, Section 1] and [5, Section 1.2] for more details.

The model is described by using two independent simple symmetric random walks X and Y. We denote by P_X^x and P_Y^y the law of X and Y starting from x and $y \in \mathbb{Z}^d$, respectively. In particular, we denote by $P_X = P_X^0$ and $P_Y = P_Y^0$. Also, we define $p_n(x) = P_X(X_n = x)$ for $x \in \mathbb{Z}^d$ and $n \in \mathbb{N}$.

For fixed Y, we define the Gibbs measure $\mu_{N,Y}^\beta$ of X by

$$\mu_{N,Y}^\beta(dX) = \frac{1}{Z_{N,Y}^\beta} P_X\left[\exp\left(\beta \sum_{k=1}^N \mathbf{1}\{X_k = Y_k\}\right) : dX\right],$$

where $\beta \geq 0$ is the inverse temperature and

$$Z_{N,Y}^\beta = P_X\left[\exp\left(\beta \sum_{k=1}^N \mathbf{1}\{X_k = Y_k\}\right)\right]$$

is the *quenched partition function*. Also, we define the *annealed partition function* by

$$P_Y\left[Z_{N,Y}^\beta\right] = P_{X,Y}\left[\exp\left(\beta \sum_{k=1}^N \mathbf{1}\left\{X_k = Y_k\right\}\right)\right],$$

where $P_{X,Y} = P_X \otimes P_Y$ is the product measure of P_X and P_Y.

Let

$$L_N(X,Y) = \sum_{k=1}^N \mathbf{1}\{X_k = Y_k\} \text{ and } L(X,Y) = \lim_{N\to\infty} L_N(X,Y) = \sum_{n\geq 1} \mathbf{1}\{X_n = Y_n\}$$

be the collision local time up to time N and the collision local time, respectively. The monotone convergence theorem implies that the following limit exists

$$Z_Y^\beta = \lim_{N\to\infty} Z_{N,Y}^\beta = P_X\left[\exp\left(\beta L(X,Y)\right)\right]$$

P_Y-almost surely and

$$P_Y\left[Z_Y^\beta\right] = \lim_{N\to\infty} P_Y\left[Z_{N,Y}^\beta\right] = P_{X,Y}\left[\exp\left(\beta L(X,Y)\right)\right].$$

We set

$$\beta_1^q(d) = \sup\left\{\beta \geq 0 : Z_Y^\beta < \infty,\ P_Y\text{-a.s.}\right\}$$

and

$$\beta_1^a(d) = \sup\left\{\beta \geq 0 : P_Y\left[Z_Y^\beta\right] < \infty\right\}.$$

Then, phase transitions occur at $\beta_1^q(d)$ and $\beta_1^a(d)$.

It is known that when X and Y are simple symmetric random walks on $\mathbb{Z}^d$,

$$\begin{aligned} &\beta_1^q(d) = \beta_1^a(d) = 0 && \text{if } d = 1,2 \\ &0 < \beta_1^a(d) < \beta_1^q(d) < \infty && \text{if } d \geq 3 \end{aligned}$$

[1,2,5,7].