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Smooth solutions to portfolio liquidation problems under price-sensitive market impact[☆]

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Abstract

We consider the stochastic control problem of a financial trader that needs to unwind a large asset portfolio within a short period of time. The trader can simultaneously submit active orders to a primary market and passive orders to a dark pool. Our framework is flexible enough to allow for price-dependent impact functions describing the trading costs in the primary market and price-dependent adverse selection costs associated with dark pool trading. We prove that the value function can be characterized in terms of the unique smooth solution to a PDE with singular terminal value, establish its explicit asymptotic behavior at the terminal time, and give the optimal trading strategy in feedback form.
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1. Introduction

Traditional financial market models assume that asset prices follow an exogenous stochastic process and that all transactions can be settled without any impact on market prices. This assumption is appropriate for small investors who trade only a negligible proportion of the average daily trading volume. It is not always appropriate, though, for institutional investors trading large blocks of shares over a short time span.

The analysis of optimal liquidation problems has received considerable attention in the mathematical finance and stochastic control literature in recent years. Starting with the paper of Almgren & Chriss [2] existence and uniqueness results of optimal liquidation strategies under various market regimes and price impact functions have been established by many authors, including [4,5,8,12–15,19–21,31,32]. One of the main characteristics of stochastic optimization problems arising in portfolio liquidation models is the singular terminal condition of the value function induced by the liquidation constraint. The singularity is already present, yet not immediately visible, in the original price impact model of Almgren & Chriss [2]. Within their mean variance framework and with arithmetic Brownian motion as the benchmark price process, the objective function is deterministic, and the optimization problem is essentially a classical variational problem where the terminal state constraint causes no further difficulties. However, when considering a geometric Brownian motion as the underlying price process as in Forsyth et al. [12], the optimal execution strategies become price-sensitive. One is then faced with a genuine stochastic control problem where the singularity becomes a challenge when determining the value function and applying verification arguments.

Several approaches to overcome this challenge have recently been suggested in the stochastic control literature. Forsyth et al. [12] solve the control problem numerically by penalizing open positions at the final time. Ankirchner & Kruse [5] characterize the value function of a Markovian liquidation problem as the unique viscosity solution to the Hamilton–Jacobi–Bellman (HJB) equation. Their verification argument uses a discrete approximation of the continuous time model. Ankirchner et al. [4], Graewe et al. [14], and Horst et al. [16] consider non-Markovian liquidation problems where the cost functional is driven by general adapted factor processes and the HJB equation solves a BSDE or BSPDE, depending on the dynamics of the factor processes. In all three cases, existence of solutions to the HJB equation is established by analyzing limits of sequences of BS(P)DEs with increasing finite terminal values while the verification argument uses generalized Itô-Kunita formulas [14], resp., the link between degenerate BSPDEs and forward–backward stochastic differential equations [16].

A general class of Markovian liquidation problems has been solved in Schied [31] by means of Dawson–Watanabe superprocess. This approach avoids the use of HJB equations. Instead, it uses a probabilistic verification argument based on log-Laplace functionals of superprocesses that requires sharp upper and lower bounds for the candidate value function.

This paper establishes existence of a smooth solution to a class of Markovian portfolio liquidation problems. While existence of a weakly differentiable solution to our HJB equation can be inferred from [14] and existence of optimal liquidation strategies can be inferred from, e.g., [4,14], smooth solutions to stochastic portfolio liquidation problems have not yet been established in the literature before. As in [31] the key is to know the precise asymptotic behavior of the value function at the terminal time. The asymptotics allows us to characterize the HJB equation in terms of a PDE with *finite* terminal value yet singular nonlinearity, for which existence of a unique smooth solution can be proved using standard fixed point arguments in a suitable function space.

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