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Functional limit theorems for a new class of non-stationary shot noise processes

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Abstract

We study a class of non-stationary shot noise processes which have a general arrival process of noises with non-stationary arrival rate and a general shot shape function. Given the arrival times, the shot noises are conditionally independent and each shot noise has a general (multivariate) cumulative distribution function (c.d.f.) depending on its arrival time. We prove a functional weak law of large numbers and a functional central limit theorem for this new class of non-stationary shot noise processes in an asymptotic regime with a high intensity of shot noises, under some mild regularity conditions on the shot shape function and the conditional (multivariate) c.d.f. We discuss the applications to a simple multiplicative model (which includes a class of non-stationary compound processes and applies to insurance risk theory and physics) and the queueing and work-input processes in an associated non-stationary infinite-server queueing system. To prove the weak convergence, we show new maximal inequalities and a new criterion of existence of a stochastic process in the space $\mathbb{D}$ given its consistent finite dimensional distributions, which involve a finite set function with the superadditive property.

Keywords: Shot noise processes; Functional weak law of large numbers; Functional central limit theorem; Gaussian limit; Non-stationarity; Skorohod J_1 topology; Weak convergence; Maximal inequalities; Criterion of existence in the space $\mathbb{D}$

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1. Introduction

We consider a class of non-stationary shot noise processes $X := \{X(t) : t \geq 0\}$ described as follows. Let $A := \{A(t) : t \geq 0\}$ be a counting process with arrival times $\{\tau_i : i \in \mathbb{N}\}$. Let $\{Z_i : i \in \mathbb{N}\}$ be a sequence of conditionally independent $\mathbb{R}^k$-valued ($k \geq 1$) random vectors given the event times $\{\tau_i : i \in \mathbb{N}\}$. For each $i \in \mathbb{N}$, the distribution of Z_i depends on τ_i only. To indicate the dependence of Z_i on τ_i explicitly, we write $Z_i(\tau_i)$ for Z_i. The regular conditional probability for $Z_i(\tau_i)$ given that $\tau_i = t$, $t \geq 0$, is given by

$$P(Z_i(\tau_i) \leq x | \tau_i = t) = F_t(x), \quad t \geq 0, \quad x \in \mathbb{R}^k, \tag{1.1}$$

where for two vectors $x = (x_1, \ldots, x_k)$, $y = (y_1, \ldots, y_k) \in \mathbb{R}^k$, $x \leq y$ means $x_i \leq y_i$ for each $i = 1, \ldots, k$ and $F_t(\cdot)$ is a joint/multivariate cumulative distribution function (c.d.f.) for each $t \geq 0$. Let $H : \mathbb{R}_+ \times \mathbb{R}^k \to \mathbb{R}$ be a deterministic measurable function representing the shot shape or the (impulse) response function. See the precise assumptions on $F_t(\cdot)$ and H in Assumption 2. Define the non-stationary shot noise process X by

$$X(t) := \sum_{i=1}^{A(t)} H(t - \tau_i, Z_i(\tau_i)), \quad t \geq 0. \tag{1.2}$$

In the literature the sequence of random variables $\{Z_i\}$ is often assumed to be i.i.d., independent of the arrival processes of shot noises (see, e.g., [9,12,13,18,26–28,31,40]). Limited work has studied for the sequence $\{Z_i\}$ with certain dependence structures. For example, in [33], $\{Z_i\}$ is modulated by a finite-state Markov chain and is conditionally independent with a distribution depending on the state of the chain at the arrival time of the shot noise (also modulated by the same chain). In [38], a cluster shot noise model is studied where $\{Z_i\}$ depends the same 'cluster mark' within each cluster. However, the 'non-stationarity' of shot noises has been neither explicitly modeled nor adequately studied, although it often occurs in stochastic systems (see, e.g., [2,8,15,35,36,45] and references therein). In our model, the sequence $\{Z_i\}$ is assumed to be conditionally independent given the arrival times and the distribution depends upon the arrival times. We have explicitly modeled "non-stationarity" in the distribution of shot noises. In addition, the arrival process is also allowed to be a general non-stationary point process.

In this paper, we establish the functional weak law of large numbers (FWLLN) and functional central limit theorems (FCLTs) for this class of non-stationary shot noise processes in an asymptotic regime where the arrival rate is large while fixing the shot noise distributions $F_t(x)$ and shot shape function H (see Assumptions 1 and 2). (It is often referred to as the "high intensity/density regime" [4,17,19,37].) Here we assume that the arrival process satisfies an FCLT with a continuous limiting process and a non-stationary arrival rate function. In the FCLT, we obtain a non-stationary stochastic process limit for the diffusion-scaled shot noise process (Theorem 2.2). The limit can be written as a sum of two independent processes, one as an integral functional of the limiting arrival process, and the other as a continuous Gaussian process. When the arrival limit is Gaussian, the limiting shot noise process becomes a Gaussian process. We also consider a finite collection of shot noise processes as defined in (1.2) with a family of shot shape functions but the same arrival process and noises, and prove their joint convergence in Theorem 2.3.

We discuss the applications of the FCLT to a simple multiplicative model and the queueing and work-input process for a non-stationary infinite-server queueing model ($G_t/G_t/\infty$) in Section 3. The simple multiplicative model requires that the shot shape function $H(t, x) =$