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A random flight process associated to a Lorentz gas with variable density in a gravitational field

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Abstract

We investigate a random flight process approximation to a random scatterer Lorentz gas with variable scatterer density in a gravitational field. For power function densities we show how the parameters of the model determine recurrence or transience of the vertical component of the trajectory. Finally, our methods show that, with appropriate scaling of space, time and the density of obstacles, the trajectory of the particle converges to a diffusion with explicitly given parameters.

MSC: 60F17

Keywords: Lorentz model; External field; Invariance principles

1. Introduction

We consider the random flight process that arises as the Boltzmann–Grad limit of a random scatterer model ("Lorentz gas") in a constant gravitational field in dimension three. We also extend our model to other dimensions, where it can be considered as the random walk approximation to the Boltzmann–Grad limit. The Lorentz gas model, which was introduced in

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1905 as a model for the motion of an electron in a metallic body [20], has been studied extensively in the mathematics and physics literature. See [9] for a recent survey. Fundamentally, the model consists of a particle moving in an array of fixed convex scatterers, which are placed either periodically or randomly, and the particle either reflects specularly off of the scatterers (hard core model) or is pushed away via a potential (soft core model). We are motivated by the three dimensional random scatterer hard core model where, in addition to interacting with scatterers, the particle is also pulled down by a constant gravitational field. We generalize the process to arbitrary dimension and investigate whether it is recurrent or transient. We show that dimension three with constant density of scatterers is critical for determining recurrence versus transience with respect to both dimension and the rate at which the density of scatterers increases.

Various aspects of the influence of a gravitational field on a Lorentz gas have previously been investigated, see e.g. [7,28,29,34]. Of this prior work, only [7] has worked directly with the Lorentz gas model. In [7] the authors establish the surprising result that the trajectory of a ball in a two-dimensional, periodic, hard core, Lorentz gas with gravitation is (neighborhood) recurrent [7, Theorem 1]. Heuristically, the pull of gravity is not strong enough to pull the particle to $-\infty$, but rather the scatterers are enough of an obstruction to make the particle bounce back up to some finite energy level infinitely often. In addition to this (neighborhood) recurrence result, a diffusive limit for the particle trajectory is also determined [7, Theorem 2]. We remark that although [7, Theorem 1] is stated with a hypothesis that the particle has a sufficiently high initial speed, as shown in [7, p. 838] all one needs for the particle to return to a fixed finite energy level infinitely often is for the initial speed to be positive (this is a slight oversimplification—in the deterministic setting of [7] the velocity must be uniformly distributed on a particular set specified in [7], but there can be an arbitrarily small upper bound on the initial speed of a particle whose initial velocity is in this set, see [7] for details).

One of the motivations of the present work is to investigate the robustness of these results under perturbations of the model. However, as the authors of [7] mention in their companion paper [6] their approach should extend to the three dimensional case, but the extension currently seems intractable due to the complicated nature of the singularities. Thus we work, as the authors of [28,29,34] do, with the Boltzmann–Grad limit of the random Poisson scatterer Lorentz gas rather than the Lorentz gas itself. Our results suggest that dimension three is the most difficult dimension and that the problem for the periodic Lorentz gas may become tractable again in dimensions four and higher. We determine criteria for the recurrence or transience of the particle trajectory for particular forms of the density of scatterers. Our methods allow us to derive several types of invariance principles in multiple scaling regimes and determine the influence of the density of scatterers on the limiting diffusion. A similar model with constant scatterer density was previously considered in [29], where diffusion limits were obtained but questions of transience and recurrence were not addressed.

The Boltzmann–Grad limit is a low density limit in which the number of scatterers in a fixed box goes to infinity while, at the same time, the size of each scatterer goes to zero in such a way that the distribution of the distance between scattering events for the tracer particle has a non-degenerate limit. When the centers of scatterers are placed according to a Poisson process and the rates are chosen appropriately, the asymptotic behavior of the moving particle is described by a Markovian random flight process [10,30,31]. The Markovian nature of the Boltzmann–Grad limit is due to the following two observations: (i) re-collisions with scatterers become unlikely as the size of each scatterer goes to zero, and (ii) the Poisson nature of the scatterer locations means that knowing the location of one scatterer does not give information about the locations of the other scatterers. Since analyzing the random Lorentz gas directly is beyond

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