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Branching random walks, stable point processes and regular variation

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Abstract

Using the theory of regular variation, we give a sufficient condition for a point process to be in the superposition domain of attraction of a strictly stable point process. This sufficient condition is used to obtain the weak limit of a sequence of point processes induced by a branching random walk with jointly regularly varying displacements. Because of heavy tails of the step size distribution, we can invoke a one large jump principle at the level of point processes to give an explicit representation of the limiting point process. As a consequence, we extend the main result of Durrett (1983) and verify that two related predictions of Brunet and Derrida (2011) remain valid for this model.

MSC: primary 60J70; 60G55; secondary 60J80

Keywords: Branching random walk; Branching process; Strictly stable; Point process; Cox process; Extreme values; Rightmost point

1. Introduction

Branching random walk on the real line can be described as follows. In the zeroth generation, one particle is born at the origin. It branches into a number of offspring particles and positions them according to a point process $\mathcal{L}$ on the real line giving rise to the first generation. Each of the particles in the first generation produces offspring and they (the offsprings) undergo displacements (with respect to the positions of their parents) according to independent copies of

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the same point process $\mathcal{L}$. The position of a particle in the second generation is its displacement translated by its parent's position. This forms the second generation, and so on. Assume further that the random number of new particles produced by a particle and the displacements corresponding to the new particles are independent. The resulting system is known as a branching random walk. Let Z_n denote the number of particles in the nth generation. Clearly $\{Z_n\}_{n\geq 0}$ forms a Galton–Watson branching process with $Z_0 \equiv 1$. We assume that this branching process is supercritical and condition on its survival.

The earliest theoretical works regarding branching random walk were done by Hammersley [21], Kingman [25] and Biggins [10]. The asymptotic properties of the left-most position in the nth generation has gathered a lot of attention specially due to the relation with extremes of two dimensional Gaussian free field and many log-correlated fields. The expected order of minimal position (left-most point) in the nth generation when the displacements are assumed to be i.i.d. and having exponential moments, was derived by Addario-Berry and Reed [1]. In a pioneering work by Aïdékon [2], the law of left-most point was derived in the dependent, light tailed setting and the limit turns out to be a shifted mixture of Gumbel distribution. Other related works dealing with the extreme positions are Hu and Shi [22], Bramson et al. [14]. In this above setting, the point process of appropriately shifted positions of the particles was studied by Madaule [27]. The limit, although lacking explicit representation satisfies certain stability properties which were predicted by Brunet and Derrida [15]. Also such point processes arise in case of branching Brownian motion (see Aïdékon et al. [3], Arguin et al. [5,6,7]) and Gaussian free field (see Biskup and Louidor [11,12,13]). We refer to the survey Arguin [4] for further references and applications of branching random walk. In the case when the displacements have regularly varying tails and i.i.d., the extreme and point process were done by Durrett [18] and Bhattacharya et al. [9] respectively.

In this article, we venture beyond the i.i.d. setting, and assume that the displacements of offspring particles coming from the same parent will be dependent and multivariate regularly varying. As in the independent case the broad aim here is to answer the following natural question: *Is it possible to describe the limiting point process of properly scaled positions of the nth generation particles?* We answer this question positively and give an explicit description of the limiting point process which forms a *randomly scaled scale-decorated Poisson point process* (see Definition 2.4). To this end, we study the stability property (as introduced by Davydov et al. [16]) of the limiting point process and relate it to the regular variation of point processes (in the sense of Hult and Lindskog [23]) based on heavy-tailed analogues of the main results of Subag and Zeitouni [34]. Our mode of proof gives a mathematical justification behind obtaining a scale-decorated Poisson point process in the limit. We also extend the result of Durrett [18] and show that, as in light-tailed case, the asymptotic position of the rightmost point is not qualitatively affected by the presence of dependence. Also the stability properties of the limiting point process show that the predictions of Brunet and Derrida [15] hold true in the heavy-tailed, dependent setup.

This article is organised as follows. In Section 2, we present the background to regular variation of point process and definition of strictly stable point processes. We also develop the notations for branching random walks and state the main results on stability (Theorem 2.3) and branching random walk (Theorem 2.6). Section 3 describes the characterisations for randomly scaled scale-decorated Poisson point process and analogous results of Subag and Zeitouni [34] and we use them to prove Theorem 2.3. In Section 4 we divide the proof of Theorem 2.6 into several small lemmas. A few important consequences of Theorem 2.6 are proved in Section 5.