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Spectrally negative Lévy processes with Parisian reflection below and classical reflection above

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Received 25 August 2016; received in revised form 20 February 2017; accepted 29 April 2017

Available online xxxx

Abstract

We consider a company that receives capital injections so as to avoid ruin. Differently from the classical bail-out settings, where the underlying process is restricted to stay at or above zero, we study the case bail-out can only be made at independent Poisson observation times. Namely, we study a version of the reflected process that is pushed up to zero only on Poisson arrival times at which the process is below zero. We also study the case with additional classical reflection above so as to model a company that pays dividends according to a barrier strategy. Focusing on the spectrally negative Lévy case, we compute, using the scale function, various fluctuation identities, including capital injections and dividends.

MSC: 60G51; 91B30

Keywords: Capital injections; Dividends; Scale functions; Lévy processes; Excursion theory

1. Introduction

In this paper, we revisit the study of risk processes where a company is bailed out by capital injections. In the classical setting, capital injections can be made at all times and instantaneously;

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http://dx.doi.org/10.1016/j.spa.2017.04.013

0304-4149/

the resulting process becomes the classical reflected process that stays at or above zero uniformly in time. In reality, however, this may not be observable continuously, and it is necessary to consider the time taken to execute this process. Motivated by recent research on Poisson observations and Parisian ruin found in (among others) [3,4,7,13], we consider the scenario where bail-outs can only be made at independent Poisson times.

We consider a general spectrally negative Lévy process as the underlying process. Additionally, we are given *Poisson observation times*, or an increasing sequence of jump times of an independent Poisson process. At each Poisson observation time when the process is below zero, the process is pushed up to zero: we call this *Parisian reflection*. Related processes with underlying compound Poisson processes have been studied in [1] and [5]. In the former study, a number of identities were obtained when solvency is only observed periodically, whereas the latter study analyzes a case where observation intervals are Erlang-distributed.

In this paper, we are also interested in the case of a dividend-paying company that pays dividends according to a barrier strategy. With regard to this, we consider a version of the (doubly) reflected process, where, given a spectrally negative Lévy process reflected from above, it is pushed up to zero at Poisson observation times at which the process is below zero.

Our objective is to obtain, using fluctuation/excursion theories, concise expressions of several identities of the following:

(1) the spectrally negative Lévy process with Parisian reflection below, and
(2) its variant with additional classical reflection above.

In particular, we are interested in the following:

(**absolute ruin**): We define *absolute ruin* to be the event that the process goes below a specified level $a < 0$. The absolute ruin probability and its time can be used to evaluate the risk of the company just like the classical ruin, which is the event that the process goes below 0.

(**capital injections**): Capital injections correspond to Parisian reflection below. We compute their total discounted values for both (1) and (2) for the infinite horizon case as well as for the cases that are killed upon exiting $[a, b]$, $[a, \infty)$ and $(-\infty, b]$ for $a < 0 < b$.

(**dividends**): If dividends are assumed to be paid continuously, then they are modeled by the classical reflection above in process (2). We compute their total expected discounted values for the infinite horizon case and for the case that is killed upon exiting $[a, \infty)$ for $a < 0$.

We use the scale function to compute these fluctuation identities. It is well known, as in [12], that the scale function existing for every spectrally one-sided Lévy process can be applied to obtain various fluctuation identities of the process and its reflected/refracted processes.

The main difficulty with the spectrally negative Lévy process is handling the possible overshoot at its down-crossing time: it is typically necessary to express the identities in terms of the convolution of the Lévy measure and the resolvent measure via the scale function. Recent results show, however, that these can be concisely written under some conditions. In this paper, for process (1), we use the simplifying formula obtained in [13]. Together with this, the desired identities for the *bounded variation case* can be obtained using a well-known technique via the strong Markov property; see, e.g., [4].

For the *unbounded variation case*, we shall use excursion theory instead of using the commonly used approximation methods as in [4]. In doing so, we first obtain an excursion-measure version of the simplifying formula in [13] (see Theorem 5.1). Using this and excursion theory, we can obtain identities directly without relying on the approximation scheme. Our