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# Dynamics of multivariate default system in random environment☆

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**Abstract**

We consider a multivariate default system where random environmental information is available. We study the dynamics of the system in a general setting of enlargement of filtrations and adopt the point of view of change of probability measures. We also make a link with the density approach in the credit risk modelling. Finally, we present a martingale characterization result with respect to the observable information filtration on the market.



*MSC:* 91G40; 60G20; 60G44

*Keywords:* Multiple defaults; Prediction process; Product space and product measure; Change of probability measure; Density hypothesis; Martingale characterization

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## 1. Introduction

We consider a system of finite default times to study their probability distributions and the dependence between the default system and the environmental market. In the credit risk analysis, the environmental information appears to be an important factor. Besides the dependence structure among the underlying defaults, we also need to investigate the role of other market information upon the system of multiple defaults, and vice versa, the impact of default events on the market. In the credit risk modelling such as in the book of Bielecki and Rutkowski [6] and the paper of Elliott, Jeanblanc and Yor [16], the information structure concerning default times is described by the theory of enlargement of filtrations. In general, we suppose that on the market which is represented by a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, the environmental information is modelled by a reference filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ and the default information is then added to form an enlarged filtration $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$ which represents the global information of the market. The modelling of the dependence structure of multiple default times is then diversified in two directions by using the bottom-up and top-down models. In the former approach, one starts with a model for the marginal distribution of each default time and then the correlation between them is made precise (see Frey and McNeil [18] for a survey). While in the top-down models which are particularly developed for the portfolio credit derivatives (see for example Arnsdorff and Halperin [4], Bielecki, Crépey and Jeanblanc [5], Cont and Minca [10], Dassios and Zhao [11], Ehlers and Schönbucher [13], Filipović, Overbeck and Schmidt [17], Giesecke, Goldberg and Ding [20], Sidenius, Piterbarg and Andersen [31] among others), we study directly the cumulative loss process and its intensity dynamics.

In this paper, we consider a multi-default system in the presence of environmental information by using a general setting of enlargement of filtrations. In order to fully investigate the different key elements in the modelling, we use one random variable $\chi$ on $(\Omega, \mathcal{A})$ valued in a polish space $E$ to describe default risks and to study the dependence between the default system and the remaining market. Given an observation filtration $(\mathcal{N}_t^E)_{t\geq 0}$ on $(E, \mathcal{E})$ with $\mathcal{E} = \mathcal{B}(E)$, the observable information associated to the default system $\chi$ is given by the inverse image filtration $(\mathcal{N}_t := \chi^{-1}(\mathcal{N}_t^E))_{t\geq 0}$ on $(\Omega, \mathcal{A})$. The global market information observed at time $t \geq 0$ is then defined by the enlargement of $\mathcal{F}_t$ as $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{N}_t$. The main advantage of this presentation is twofold. First, the setting is general and can be applied flexibly to diverse situations by suitably choosing the default variable $\chi$ and the observation filtration. For example, for a multi-default system, we can include both ordered (which corresponds to top-down models) and non-ordered (which corresponds to bottom-up models) default times. Second, this framework allows us to distinguish the dependence structures of different nature, notably the correlation within the default system characterized by the so-called prediction process and the dependence between the default system and the environmental market described by a change of probability measure.

The prediction process has initially been introduced in the reliability theory (see for example Norros [29] and Knight [27]) which is defined as the conditional law of $\chi$ with respect to its observation history $\mathcal{N}_t$ and describes the dynamics of the whole default system upon each default event. When we take into account the environmental information $\mathcal{F}_t$, the market information is represented by an enlarged filtration. The main idea is to characterize the dependence between the multi-default system and the remaining market by using a change of probability method with respect to the product probability measure under which the multi-default system $\chi$ and the environmental information $\mathcal{F}_t$ are independent. In this setting, the dependence structure between the default system and the market environment under any arbitrary probability measure can be described in a dynamic manner and represented by the Radon–Nikodym derivative process

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