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Stochastic Burgers equation from long range exclusion interactions

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Abstract

We consider one-dimensional exclusion processes with long jumps given by a transition probability of the form $p_n(\cdot) = s(\cdot) + \gamma_n a(\cdot)$, such that its symmetric part $s(\cdot)$ is irreducible with finite variance and its antisymmetric part is absolutely bounded by $s(\cdot)$. We prove that under diffusive time scaling and strength of asymmetry $\sqrt{n}\gamma_n \to_{n\to\infty} b \neq 0$, the equilibrium density fluctuations are given by the unique energy solution of the stochastic Burgers equation.

MSC: 60K35

Keywords: Stochastic Burgers equation; KPZ equation; Long range exclusion; Equilibrium fluctuations; Universality

1. Introduction

Scaling limits of one-dimensional, stochastic interface models and their relation with the Kardar–Parisi–Zhang (KPZ) [23] equation

$$\partial_t \mathscr{X}_t = \frac{1}{2}\sigma^2 \Delta \mathscr{X}_t + \lambda\big(\nabla \mathscr{X}_t\big)^2 + \nu\xi$$

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have been the subject of intense study in the mathematical physics community during the last few years. In this equation, ξ stands for a standard, space–time white noise. One line of research corresponds to what is known as the *weak universality* of the KPZ equation. Roughly speaking, the KPZ equation should arise as the scaling limit of any one-dimensional, non-reversible family of stochastic interface models with local interactions on which there is a natural way to scale down the asymmetry of the model as the scale grows.

From the point of view of interacting particle systems, it is more natural to look at the derivative of the interface. The stochastic process obtained in this way corresponds to a non-reversible particle system with a *conservation law*, most of the times the number of particles of the system. This conservation law is equivalent to the local character of the dynamics in the interface model. The limiting equation will be the stochastic Burgers equation

$$\partial_t \mathscr{Y}_t = \tfrac{1}{2}\sigma^2 \Delta \mathscr{Y}_t + \lambda \nabla \mathscr{Y}_t^2 + \nu \nabla \xi, \tag{1.1}$$

where ξ is a space–time white noise and σ^2, λ, ν are model-dependent constants. This equation is the one satisfied by the space derivative of the solution of the KPZ equation.

The first mathematically rigorous work showing the convergence of the density fluctuations of an interface growth model is the one of Bertini and Giacomin [2]. The authors considered the weakly asymmetric simple exclusion process (WASEP) and they proved that its density fluctuations converge to the so-called *Cole–Hopf* solution of (1.1). Their result holds for a large class of initial conditions and it was also the first article where a proper mathematically rigorous definition of the KPZ/Burgers equation was given. Interpreted in terms of interface models, in [2] the authors proved that the scaling limit of *current fluctuations* in the WASEP is given by the KPZ equation.

The results of [2] heavily rely on the so-called *Gartner transform* (see [11,13]), which is a discrete version of the Cole–Hopf transform that effectively linearizes the asymmetric simple exclusion process (ASEP). This linearization is a blueprint of the richer integrable structure of the ASEP. This integrable structure was discovered by relating the ASEP to the Bethe Ansatz [31,34]. The theory of MacDonald processes [4] allows to discover various other systems with rich integrable structure, as well as to extend the integrable structure of ASEP [5].

In order to prove convergence of density fluctuations to solutions of (1.1), there are two well developed strategies, which are very different and complementary in nature. One strategy is to consider a model with enough structure to allow a linearizing transformation, similar to Gartner transform. If this strategy is successful, the original method of Bertini and Giacomin [2] should work, modulo model-dependent technical points. This strategy has been performed by instance in [1,8,7,6,26]. When this strategy works, the underlying integrable structure usually allows to obtain fine details of the solutions of the KPZ equation starting from special initial conditions, as well as to allow very general initial data.

The second strategy relies on the concept of *energy solutions* of the KPZ/Burgers equation (1.1), introduced in [14]. Conditioned on the uniqueness of stationary solutions of this equation, the authors proved in [14] convergence of the density fluctuations to this stationary solution for a large class of weakly asymmetric stochastic models. In particular, no integrability assumptions are made. The convergence of interface fluctuations of the associated growth models is also obtained as a consequence of the convergence of current fluctuations. Uniqueness of stationary energy solutions of the KPZ/Burgers equations as defined in [18] were proved to hold on the torus in [20] and on the real line in [10], closing the gap in the proof of convergence of [14]. In [16], the proof of convergence was generalized to include particle systems without product

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