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Fundamental accuracy of time domain finite element method for sound-field analysis of rooms

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ABSTRACT

This paper presents an assessment of the accuracy and applicability of a time domain finite element method (TDFEM) for sound-field analysis in architectural space. This TDFEM incorporates several techniques: (1) a hexahedral 27-node isoparametric acoustic element using a spline function; (2) a lumped acoustic dissipation matrix; and (3) Newmark time integration method with an absolute diagonal scaled COCG iterative solver. Sound fields in an irregularly shaped reverberation room of 166 m^3 are computed using TDFEM. The computed values and measured values for 125–500 Hz are compared, revealing that the fine structure of the computed band-limited impulse responses agree with measured ones up to 0.1 s, with a cross-correlation coefficient greater than 0.93. The cross-correlation coefficient decreases gradually over time, and more rapidly for higher frequencies. Moreover, the computed decay curves, and the reverberation times, agree well with the respective measured ones, and with a better fit the higher the frequency (up to 500 Hz).

1. Introduction

Numerical analysis methods based on wave acoustics (wave-based methods) such as finite element method (FEM) and boundary element method are powerful numerical methods used to predict sound fields in architectural space accurately with complicated boundary conditions. Generally, a wave-based method entails large computational cost for analyzing sound fields in architectural space with practical dimensions as well as practical frequency ranges. However, recently, the application of wave-based methods is increasing gradually along with the rapid progress of computer technology.

Although both time and frequency domain analyses using wave-based methods are conducted for predicting sound fields in architectural spaces, time domain analysis is straightforward in calculating the impulse response of the space in a time domain without the use of inverse Fourier transform. Various acoustical parameters used for acoustical quality evaluation are calculable from the impulse response.

The finite-difference time-domain (FDTD) method is a widely used method of computing the impulse response in a space [1–3]. The explicit formulation makes the computation efficient, and the computational effort increases linearly in direct relation to the number of discretization cells. However, the architectural space boundary shape is approximated by a staircase approximation in many cases, which is a disadvantage of the FDTD method because a room's shape is an important factor to determine acoustic properties in an architectural space.

Because it enables modeling of complicated boundary shape, FEM is an attractive method. Various methods incorporating FEM are used for computing the impulse response in a space: indirect method with inverse Fourier transform of frequency domain response computed using frequency domain FEM (FDFEM) [4], a method using FEM with a modal analysis approach [5,6], and a direct method using time domain FEM (TDFEM) with direct time integration method [7,8].

Among them, TDFEM can treat time variation of the boundary condition and medium. Although this method typically requires a solution technique of a linear system of equations at each time step, the matrices arising from the finite element formulation are generally sparse. Therefore, using a Krylov subspace iterative solver such as the Conjugate Gradient (CG) method, the memory and operations required for TDFEM can be reduced approximately to the order of degrees of freedom (DOFs) of the finite element model, if a fast convergence of the iterative solver can be expected. In addition, using the unconditionally stable direct time integration method, TDFEM has unconditional stability. Therefore, if TDFEM provides a similarly accurate result as the FDTD method using fewer elements per wavelength and a much larger time

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interval, which engenders a decrease of DOFs and total number of time steps compared to those required for the FDTD method, then the computational costs for the TDFEM would be favourable compare to those required for the FDTD method. Moreover, TDFEM is well suited to vector and/or parallel computation, which is also the case for the FDTD method. Nevertheless, few studies have examined application of TDFEM for sound-field analyses in architectural spaces.

To reduce the computational cost of the TDFEM and to use a method for practical situations such as the design process of a room and acoustic improvement of an existing room, the authors developed TDFEM, including the following techniques [9]: (1) hexahedral 27-node isoparametric acoustic element using spline function [10,11], (2) lumped acoustic dissipation matrix, and (3) Newmark time integration method with absolute diagonal scaled COCG Krylov subspace iterative solver. Moreover, the authors have already shown that the linear system of equations can be solved efficiently using the iterative solver [9,12].

However, accuracy and applicability of the TDFEM to sound-field analyses in architectural space remain unclear. Consequently, this study was undertaken to assess the basic accuracy and applicability of the TDFEM for sound-field analysis in architectural space through comparison with measured values.

This paper is organized as follows. Section 2 presents outlines of both mathematical and physical bases of the presented TDFEM. In Section 3, we present a comparison of computed and measured results of a sound field in an irregularly shaped reverberation room with volume of 166 m^3. Finally, in Section 4, we present concluding remarks.

2. Sound-field analyses using time domain finite element method: TDFEM

Following a standard finite element procedure based on the principle of minimum total potential energy applied to a three-dimensional sound field, the following discretized matrix equation in the frequency domain is obtained.

$$[K]\{p\} + \mathrm{i}\omega[C]\{p\} - \omega^2[M]\{p\} = \mathrm{i}\omega\rho v_0\{W\} \tag{1}$$

Therein, $[M]$, $[C]$, and $[K]$ respectively denote acoustic mass, dissipation, and stiffness matrices. Furthermore, i, $\{p\}$, ρ, ω, v_0, and $\{W\}$ respectively signify an imaginary unit ($i^2 = -1$), sound pressure vector, the air density, angular frequency, the particle velocity and distribution vector. Assuming that $\cdot$ and $\cdot\cdot$ respectively signify first-order and second-order derivatives with respect to time, the semi-discrete equation in the time domain can be expressed as presented below.

$$[M]\{\ddot{p}\} + [C]\{\dot{p}\} + [K]\{p\} = \rho\dot{v}_0\{W\} \tag{2}$$

Using an interpolation function $N(x,y,z)$, the sound pressure $p(x,y,z)$ on an arbitrary point at (x,y,z) is assumed to be

$$p(x,y,z) = \{N(x,y,z)\}^T\{p\}_e. \tag{3}$$

The analysis described in this paper uses a hexahedral 27-node isoparametric acoustic element with the spline polynomial function for $N(x,y,z)$ [10,11]. Acoustic element matrices used to construct global matrices in Eq. (1) are given as expressed below.

$$[K]_e = \int_e \left(\left\{\frac{\partial N}{\partial x}\right\}\left\{\frac{\partial N}{\partial x}\right\}^T + \left\{\frac{\partial N}{\partial y}\right\}\left\{\frac{\partial N}{\partial y}\right\}^T + \left\{\frac{\partial N}{\partial z}\right\}\left\{\frac{\partial N}{\partial z}\right\}^T \right) dV, \tag{4}$$

$$[M]_e = \frac{1}{c^2}\int_e \{N\}\{N\}^T dV, \tag{5}$$

$$[C]_e = \frac{1}{c}\int_{e'} \frac{1}{z_n}\{N\}\{N\}^T dS. \tag{6}$$

Herein, c and z_n respectively represent the speed of sound and normal surface impedance; e' denotes the surface area to be integrated. As described in this paper, a locally reactive model is assumed for dissipation; then, $[C]_e$ can be reassembled into a diagonal matrix as the lumped acoustic dissipation matrix.

In the time domain, Newmark β method [13] is used to solve Eq. (2) step by step. If $\{p\}_t, \{\dot{p}\}_t$ and $\{\ddot{p}\}_t$ at time t are known, then $\{p\}_{t+\Delta t}$ and $\{\dot{p}\}_{t+\Delta t}$ can be given as

$$\{p\}_{t+\Delta t} = \{p\}_t + \Delta t\{\dot{p}\}_t + (\Delta t)^2\left(\frac{1}{2} - \beta\right)\{\ddot{p}\}_t + (\Delta t)^2\beta\{\ddot{p}\}_{t+\Delta t}, \tag{7}$$

$$\{\dot{p}\}_{t+\Delta t} = \{\dot{p}\}_t + \Delta t(1-\gamma)\{\ddot{p}\}_t + \Delta t\gamma\{\ddot{p}\}_{t+\Delta t}. \tag{8}$$

In those equations, Δt is the time interval between t and $t + \Delta t$, and γ, β are parameters related to the accuracy and stability of the method. For this study, γ is set to 1/2 to maintain the second-order accuracy. Then, with the Eqs. (7) and (8), it is possible to transform Eq. (2) into

$$\left[[M] + \frac{\Delta t}{2}[C] + \beta(\Delta t)^2[K]\right]\{\ddot{p}\}_{t+\Delta t} = \{f\}_{t+\Delta t} - [C]\{P\} - [K]\{Q\}, \tag{9}$$

where

$$\{P\} = \{\dot{p}\}_t + \frac{\Delta t}{2}\{\ddot{p}\}_t,$$
$$\{Q\} = \{p\}_t + \Delta t\{\dot{p}\}_t + \left(\frac{1}{2} - \beta\right)(\Delta t)^2\{\ddot{p}\}_t. \tag{10}$$

Consequently, $\{\ddot{p}\}_{t+\Delta t}$ is calculable by solving Eq. (9). Finally, the sound pressure $\{p\}_{t+\Delta t}$ and first derivative of the sound pressure $\{\dot{p}\}_{t+\Delta t}$ are calculable respectively by substituting $\{\ddot{p}\}_{t+\Delta t}$ into Eqs. (7) and (8).

Several Newmark methods exist with different values of parameter β [14]. The following three special cases of the Newmark methods are well known: constant average acceleration method with $\beta = 1/4$, linear acceleration method with $\beta = 1/6$, and Fox–Goodwin method with $\beta = 1/12$. Here, if $\beta \geqslant 1/4$, then the Newmark method is unconditionally stable.

Moreover, the Newmark method usually requires solution of the linear system of equations with Eq. (9) at each time step. As a shorter expression, Eq. (9) can be rewritten as

$$[A]\{\ddot{p}\}_{t+\Delta t} = \{b\}_{t+\Delta t}. \tag{11}$$

Therein, $[A]$ and $\{b\}_{t+\Delta t}$ respectively denote the coefficient matrix and the right-hand side vector in Eq. (9). When z_n is given as a complex number, $[A]$ becomes a complex sparse symmetric matrix – a non-Hermitian – because $[K]$ and $[M]$ are real sparse symmetric matrices; $[C]$ is a complex diagonal matrix. As a special case, when z_n is given as a real number for simplification, $[A]$ becomes a real symmetric matrix. Then, to solve Eq. (11), we adopt Conjugate Orthogonal Conjugate Gradient (COCG) method [15], which is the efficient iterative solver for the FEM formulation in [9], and which is also used here. This method is equivalent to CG method when $[A]$ becomes a real symmetric matrix. As a preconditioning technique, the absolute diagonal scaling [16], not increasing the number of complex component in the $[A]$, is used. Using preconditioning, the scaled matrix $[A]$ is written as

$$a'_{ij} = \frac{a_{ij}}{\sqrt{|a_{ii}|}\sqrt{|a_{jj}|}}, \tag{12}$$

where a_{ij} and a'_{ij} are components of $[A]$ before and after preconditioning. This preconditioned COCG method is well suited to parallelization. Using the relative residual 2-norm δ, the stopping criterion for stopping the succession of iterations of the COCG method is used as