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Boscovich's geometrical principle of continuity, and the "mysteries of the infinity"

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Abstract

In this paper we give a detailed account of Boscovich's geometrical principle of continuity. We also compare his ideas with those of his forerunners and successors, in order to cast some light on his possible sources of inspiration and to underline the elements of novelty in his approach to the subject.

Sommario

In questo lavoro si presenta in modo dettagliato il principio di continuità geometrica di Boscovich. Si confrontano le sue idee con quelle dei suoi precursori e successori, con l'obiettivo di far luce sulle sue possibili fonti di ispirazione e sottolineare gli elementi di novità del suo approccio all'argomento.

MSC: 01A50; 01A45; 01A55

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1. Introduction

The law, or principle, of (geometrical) continuity can be synthetically enounced as follows: let a figure be conceived to undergo a certain *continuous* variation, and let some *general* property concerning it be granted as true, so long as the variation is confined *within* certain limits; then the same property will belong to *all* the successive states of the figure (that is, all the states which admit the property being expressed), the enunciation being *modified* (occasionally) according to known rules.[1] This principle has, to some extent,

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[1] See [Poncelet, 1822, xiii].

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been accepted and employed in all ages of the history of mathematics, but it is only in modern times that its validity as a means of discovery or proof has been achieved. In fact, although many results of this principle are to be found in the writings of Gaspard Monge, and the members of his "school", only after Jean-Victor Poncelet's *Traité des propriétés projective des figures* did it come to be universally acknowledged [Poncelet, 1822].

As will be seen in section 2, the first germ of the principle of continuity is to be found in the *principle of analogy* introduced by Johannes Kepler, without any explicit enunciation, in his study *De coni sectionibus*, included in chapter IV of *Ad Vitellionem paralipomena quibus astronomiae pars optica traditur* [Kepler, 1604, 92–96]. As introduced by Kepler, the principle of continuity was largely independent from algebraic considerations, although the consideration of the behaviour of certain functions in limiting conditions was fundamental to his study. Only later were developments suggested by the occurrence of negative and imaginary roots in second degree equations applied to geometry.

Traces of the principle also appear in Girard Desargues' *Brouillon project* [Desargues, 1639], in some works of Wilhelm Gottfried Leibniz, Isaac Newton and other authors of the eighteenth century.

The earliest thorough treatment of this subject is found in two of Roger Joseph Boscovich's works, the dissertation *De transformatione locorum geometricorum ubi de continuitatis lege, ac de quibusdam infiniti mysteriis*, included in the third volume of his *Elementa universae matheseos* [Boscovich, 1754b], and the dissertation *De continuitatis lege* [Boscovich, 1754c]. With these essays, Boscovich wanted to cast light on the role of the principle of continuity in natural sciences and in geometry, and to raise it to the rank of a general method, aiming to make it the starting point for his major work *Theoria philosophiae naturalis* [Boscovich, 1758]. In the first essay, he proceeded by geometrical reflections, and in the second one, also through the analysis of certain natural phenomena, he stressed the philosophical aspects of the principle.

The mechanical applications of the principle of continuity and its philosophical features, which also involve the concept of *continuous*, as developed in the dissertation *De continuitatis lege*, have been widely investigated.[2] On the contrary, although there are some contributions concerning Boscovich's principle of geometrical continuity, as expounded in the dissertation *De transformatione locorum geometricorum*,[3] an in-depth study of Boscovich's ideas, and achievements, on this topic still seems to be lacking.

The goal of this paper is to give an account of Boscovich's approach to the principle of geometrical continuity through a detailed analysis and discussion of the eleven rules ("canones"), by which he operated in the transformation of geometrical loci. Then, by illustrating and commenting a series of relevant examples, among the many offered by Boscovich in order to validate the rules, we penetrate the "mysteries of infinity", as he called certain phenomena which arise in geometry when some quantity grows beyond any limit.[4] Moreover, we aim to cast some light on his possible sources of inspiration and, in order to underline the novel elements of some of Boscovich's geometrical ideas, to investigate to what extent his achievements anticipated certain concepts later introduced by Lazar Carnot and Poncelet.

Our paper expands and completes the pioneering works of Taylor, and Manara and Spoglianti.

We end this introduction with a biographical note on Boscovich.[5]

[2] See [Boscovich, 1961, 1991, 1993] and, in particular, [Stipanić, 1975, 1993], [Homann, 1993], [Schubring, 2005, 179–182], [Guzzardi, 2014], [Martinović, 2015], and the reference quoted there.

[3] See [Taylor, 1881, lxxiii–lxxviii], [Manara and Spoglianti, 1979], [Martinović, 1991].

[4] In a letter to G.S. Conti (written in Italian), on February 26th, 1762, quoted by Guzzardi [2014, 24–25], Boscovich explained what he meant by "mysteries of infinity": "Everywhere… the infinity enters, our limited and finite mind is lost, because our ideas are too weak to clearly conceive it. Therefore, these I call *mysteries of infinity*, and I distinguish them from the absurdies, as I found in the actual extension of an absolutely infinite line. The absurdies make me believe a thing is impossible; the mysteries, the difficulties of conceiving, the clouds that veil our imagination, only make me think of the weakness of our mind." This locution will later be cleared through the discussion of some examples.

[5] For more details see for instance [Hill, 1961], [Casini, 1971], for an account of his entire work see [Proverbio, 2007], for his mathematical genealogy and work see [Pepe, 2010a, 2010b], and for his conception of matter [Guzzardi, 2014].

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