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A novel method for importance measure analysis in the presence of epistemic and aleatory uncertainties

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Abstract For structural systems with both epistemic and aleatory uncertainties, research on quantifying the contribution of the epistemic and aleatory uncertainties to the failure probability of the systems is conducted. Based on the method of separating epistemic and aleatory uncertainties in a variable, the core idea of the research is firstly to establish a novel deterministic transition model for auxiliary variables, distribution parameters, random variables, failure probability, then to propose the improved importance sampling (IS) to solve the transition model. Furthermore, the distribution parameters and auxiliary variables are sampled simultaneously and independently; therefore, the inefficient sampling procedure with an "inner-loop" for epistemic uncertainty and an "outer-loop" for aleatory uncertainty in traditional methods is avoided. Since the proposed method combines the fast convergence of the proper estimates and searches failure samples in the interesting regions with high efficiency, the proposed method is more efficient than traditional methods for the variance-based failure probability sensitivity measures in the presence of epistemic and aleatory uncertainties. Two numerical examples and one engineering example are introduced for demonstrating the efficiency and precision of the proposed method for structural systems with both epistemic and aleatory uncertainties.

1. Introduction

It is a common practice to analyze the impact of input uncertainty on the structural systems in reliability engineering. Generally, two different uncertainty sources, i.e., aleatory uncertainty and epistemic uncertainty, are involved.[1] Aleatory uncertainty describes the inherent variability associated with a structural system, which is referred to as irreducible, objective uncertainty. Epistemic uncertainty results from the lack of knowledge of fundamental phenomena and is related to our ability to understand, measure, and describe the systems. In most cases, there are three different theories which have been used to handle epistemic uncertainty, namely random theory, intervals theory, and fuzzy set theory.[2] In our work, we primarily focus on the random theory, which involves the probabilistic approach to represent the epistemic uncertainty.

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Sensitivity analysis, which provides the academic reference for designing, is an important part of reliability design. It can be classified into two groups, local sensitivity analysis and global sensitivity analysis (or importance measure analysis).[3] Local sensitivity analysis (LSA) is often carried out in the form of derivative of the model output with respect to the input parameters. Global sensitivity analysis (GSA) focuses on the output uncertainty over the entire range of the inputs. The limitation of LSA, as a derivative-based approach, lies in that derivatives are only informative at the base point where they are calculated and do not provide an exploration of the rest of the input spaces. This drawback will turn severe for nonlinear models. GSA, on the other hand, explores the whole space of the input factors, thus is more informative and robust than estimating derivatives at a single point of the input space. Obviously, GSA has a greater potential in engineering applications. In reliability engineering, sometimes probability distributions of the inputs cannot be known precisely which are subject to epistemic uncertainty (or the distributional parameters uncertainty).[4] Therefore, an essential issue in structural reliability analysis is how to explore the effect of epistemic uncertainty on the failure probability. To measure the effects of aleatory uncertainty of input variables on the model outputs, Refs.[5,6] established the relationship of distributional parameters and the model output characteristic value under the condition of a reference value of parameter.

A key concern in performing importance analysis would be to improve the computational efficiency, i.e., to obtain results with a modest number of model evaluations. Kriging method is adopted by Li et al.,[7] which selects the important samples to failure probability to establish the surrogate model. According to the work by Wang et al.,[8] surrogate model is used to fit the relationship between parameters and the statistical characteristics of model output. As a matter of fact, the "triple-loop" nested sampling procedure of the importance analysis can be transformed into the "double-loop". Due to the advantage of the SDP technique and point estimates method, the method adopted by the Li[9,10] can be applied to estimating the effect of aleatory uncertainty on the failure probability efficiently.

Recently, Sankararaman and Mahadevan[11] offered a novel sight of separating the aleatory and epistemic uncertainties in a system. By introducing a uniformly distributed auxiliary variable, which is statistically independent from the distribution parameters, the effect of aleatory and epistemic uncertainties on the statistical characteristics of output can be completely separated. Owing to operating without nested sampling procedure based on distribution parameters,[12,13] the method is more efficient with enough accuracy.

The objective of this work is to extend this approach to obtain importance measure analysis of failure probability in the presence of epistemic and aleatory uncertainties by using the improved importance sampling (IS) technique. The main idea of the research is firstly to establish a novel deterministic transition model for auxiliary variables, distribution parameters, random variables, failure probability. Then, propose the improved importance sampling to solve the computational model. Furthermore, the distribution parameters and auxiliary variables are sampling together simultaneously and independently. Subsequently, the improved method, which is based on importance sampling technique, combines the proper estimates to obtain the variance-based failure probability sensitivity analysis result. To take advantage of the virtues of the fast convergence of the estimates and efficient searching capability in the interesting region of the IS, the proposed method is more efficient with enough accuracy.

The remainder of the paper is organized as follows. Section 2 reviews the definitions of importance measure analysis of failure probability. This is followed by a distinct introduction of establishing the deterministic transition model for auxiliary variables, distribution parameters, random variables, failure probability in Section 3. In Section 4, the improved importance sampling technique is applied for the deterministic model, and further utilized to calculate the importance measure of failure probability. Several examples are presented to test the proposed method in Section 5. Section 6 concludes the present work with a summary.

2. Importance measure of failure probability

2.1. Description of epistemic and aleatory uncertainties

Suppose the model is denoted by $Y = g(\boldsymbol{X})$, where Y is the model output, and $\boldsymbol{X} = [x_1, x_2, \ldots, x_n]$ (n is the number of inputs) is the set of uncertain inputs. The uncertainties of the inputs are represented by probability distributions $f_X(\boldsymbol{x})$ and $\boldsymbol{x}$ is an observation of $\boldsymbol{X}$. As the model contains both aleatory and epistemic uncertainties, the performance function of the model can be given as

$$Y = g(\boldsymbol{X}|\boldsymbol{\theta}) \tag{1}$$

where $\boldsymbol{\theta}$ is the vector of epistemic parameters. Generally, there are three models which have been used to handle the epistemic uncertainty, i.e., the random model, fuzzy set model and non-probability convex set model.[14] In this paper, probabilistic theory is employed. The uncertainty of variables $\boldsymbol{X}$ can be represented by the conditional probability density function (PDF) $f_X(\boldsymbol{x}|\boldsymbol{\theta})$ with given distribution parameters $\boldsymbol{\theta}$. The uncertainty of distribution parameters $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_m]$ (m is the number of parameters) can be represented by the PDF $f_\theta(\boldsymbol{\theta})$ in this work.

In structural importance analysis, one major task is to obtain the probabilistic information of the output response Y or the probability of $Y \leqslant y$. The failure domain of the structural system is

$$F = \{\boldsymbol{X} : g(\boldsymbol{X}) < 0\} \tag{2}$$

Suppose the indicator function of this failure domain is denoted by $I_F(\boldsymbol{X})$ (or I_F), i.e.,

$$I_F = \begin{cases} 1 & \boldsymbol{X} \in F \\ 0 & \boldsymbol{X} \notin F \end{cases} \tag{3}$$

Then the failure probability of system in the presence of epistemic and aleatory uncertainties can be expressed as

$$P_f = \int_F f_X(\boldsymbol{x}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{\theta} = \int_{R^{n+m}} I_F(\boldsymbol{x}|\boldsymbol{\theta}) f_X(\boldsymbol{x}|\boldsymbol{\theta})\mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{\theta} \tag{4}$$

where R^{n+m} denote n-dimensional input variables and m-dimensional parameters.

2.2. Importance measure of failure probability

Importance analysis, which aims at exploring how the structural failure probability is affected by the epistemic and aleatory uncertainties, is an effective way to enhance the safety