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Notes and Sources

Julius Weisbach's pioneering contribution to orthogonal linear regression (1840)

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Abstract

Orthogonal linear regression is a standard statistical method which is used to fit a line to a scatter plot of data points (x_i, y_i) in situations where both variables have errors. Until now the US American R.J. Adcock has been considered to be the first who published this method, which is based on the method of least squares. We show that Julius Weisbach, professor of mathematics and engineering at Bergakademie Freiberg in Saxony (Germany) had already published in 1840 a paper in which the method is fully described and applied to an interesting problem.

We discuss the context of his discovery in order to understand the type of problems mining surveyors faced in that time and why the use of this method was found to be relevant in geodesy. Weisbach's method of solution is then explained in all detail. We show that he implicitly used the method of least squares, but presented the solution in terms of geometrical arguments adapted to the readership of the journal in which he published.

Zusammenfassung

Die orthogonale lineare Regression ist eine Standardmethode der Statistik, mit deren Hilfe man eine Ausgleichsgerade an eine Punktwolke von Datenpunkten (x_i, y_i) anpasst, wenn beide Variablen fehlerbehaftet sind. Bis jetzt wurde der US-Amerikaner R.J. Adcock als der erste angesehen, der diese Methode publiziert hat, die auf der Methode der kleinsten Quadrate beruht. Wir zeigen, dass Julius Weisbach, Professor der Mathematik und Bergmaschinenlehre an der Bergakademie Freiberg, bereits 1840 einen Artikel publizierte, in dem die Methode genau beschrieben wird und auf ein interessantes Problem angewendet wird.

Wir diskutieren den Kontext seiner Entdeckung, ein geodätisches Problem bei der Vermessung von Bergwerken. Wir beschreiben dann Weisbachs Vorgehensweise in allen Einzelheiten. Wir zeigen, dass er die Methode der kleinsten Quadrate genau kannte, aber im Interesse des Verständnisses seiner Leser geometrische Argumente benutzte.

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1. Introduction

A standard problem of applied statistics is the fit of a line

$$y = a + bx \tag{1}$$

to a scatter plot of n data points (x_i, y_i). Every student of a course in mathematical statistics is taught the technique and every statistics textbook discusses the problem and presents its solution, which is a standard example for the application of the method of least squares. The aim is usually to quantify and simplify a stochastic relationship between two random variables.[1]

Today, the standard method for solving this problem is the method of least squares that was developed independently by Legendre and Gauss at the beginning of the 19th century.[2] However, these famous mathematicians, as well as their contemporaries, were more interested in solutions of geometrical problems of astronomy and geodesy than in the approximation of functional relations. On the contrary, the study of "functional" relationships dominates in today's statistical literature. More specifically, it is the application of *ordinary least squares* that is usually taught, where the sum of squares $(y_i - a - bx_i)^2$ is minimized. This means that the vertical distances of the data points to the line are considered, the distances on lines parallel to the y-axis. The case where the orthogonal (perpendicular) distances of the data points from the line are considered, called *orthogonal least squares* or *total least squares*, is treated much less frequently in the literature. This nevertheless remains relevant in the context of geometrical problems, where the line is considered as a special geometrical curve and the points (x_i, y_i) are geometrical objects, as well as for functional relations if both variables x and y have measurement errors.

An important example of a geodetic problem in which an approximating line had to be determined appeared already in the 18th century in the field of mine surveying, i.e. geodesy applied in mining. This is discussed in detail in (Morel, 2015) in the context of a general description of the role of mathematics in the early development of the Technical Universities of Saxony.[3] The task to be solved was to determine the main strike direction of an ore vein. After various attempts based on graphical and later arithmetical methods, the mathematician and engineer Julius Weisbach (1806–1871) proposed a solution to this problem by means of the method of least squares in a paper published in 1840.[4]

In the present article we show that Weisbach indeed used orthogonal regression and obtained correct and elegant results. He was the first to publish these formulas and even an application of his method. More importantly, the genesis of this important statistical method is thus embedded in the context of early 19th century mining engineering, although it had been until now attributed to R.J. Adcock in (Adcock, 1878), whose motivations, collaborations and scientific background are unknown.

The first section of this paper is a short exposition of orthogonal linear regression as it is used today. We then turn to the context of Weisbach's work, introducing both the mathematician and the peculiar context

[1] Classical examples concern the relationship between height and weight of humans or the relation between the heights of parents and their children.

[2] See the discussion in (Stigler, 2003, 15) and (Farebrother, 1999, 53).

[3] See (Morel, 2015, 138–152).

[4] The word "proposed" is used purposely, instead of "discovered" since it is quite possible that Gauss, Legendre or another scientist of the time used orthogonal linear regression but did not publish it. Similarly (Gerling, 1843) did not mention the line fitting problem in his overview of the least square applications in geodesy.

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