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Tangent-impulse transfer from elliptic orbit to an excess velocity vector

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Abstract The two-body orbital transfer problem from an elliptic parking orbit to an excess velocity vector with the tangent impulse is studied. The direction of the impulse is constrained to be aligned with the velocity vector, then speed changes are enough to nullify the relative velocity. First, if one tangent impulse is used, the transfer orbit is obtained by solving a single-variable function about the true anomaly of the initial orbit. For the initial circular orbit, the closed-form solution is derived. For the initial elliptic orbit, the discontinuous point is solved, then the initial true anomaly is obtained by a numerical iterative approach; moreover, an alternative method is proposed to avoid the singularity. There is only one solution for one-tangent-impulse escape trajectory. Then, based on the one-tangent-impulse solution, the minimum-energy multi-tangent-impulse escape trajectory is obtained by a numerical optimization algorithm, e.g., the genetic method. Finally, several examples are provided to validate the proposed method. The numerical results show that the minimum-energy multi-tangent-impulse escape trajectory is the same as the one-tangent-impulse trajectory.

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1. Introduction

The two-body orbital transfer problem from a parking orbit to a given excess velocity vector is a fundamental one in space exploration. The minimum-energy trajectory optimization for this problem has been studied for many years. For the initial circular orbit, an approximate analytical solution was obtained for the minimum-energy three- and four-impulse transfers between a given circular orbit and a given hyperbolic velocity vector at infinity.[1] For the transfer from the initial circular orbit to an excess velocity vector, the two-impulse escape is never simultaneously of lower cost than either the one- or three-impulse.[2] Recently, Ocampo et al.[3,4] studied the one- and three-impulse escape trajectories, which can be constructed to serve as initial guesses for determining constrained optimal multi-impulsive escape trajectories. For the initial elliptic orbit, the optimal three-impulse transfer between an elliptic orbit and an escape asymptote was solved.[5] Moreover, a simple numerical technique was proposed to minimize the time-of-flight for the multi-impulse transfer from an arbitrary elliptic orbit to a hyperbolic escape asymptote.[6]

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The existing methods for the impulse escape trajectory optimization problem do not include any constraint on the impulse direction.[7,8] If the impulse direction is aligned with the velocity vector, the impulse is called "tangent impulse" and a speed change will finish the impulse maneuver to nullify the relative velocity. The tangent orbit problem has also existed for many years. The classical Hohmann transfer is a cotangent transfer and is the minimum-energy one among all the two-impulse transfers between coplanar circular orbits and between coplanar coaxial elliptic orbits.[9] The cotangent transfer problem between coplanar noncoaxial elliptic orbits has aroused considerable interest in recent years. The numerical solution was obtained based on the orbital hodograph theory.[10] Moreover, the closed-form solution was obtained by using the geometric characteristics[11] and by the flight-direction angle,[12] respectively. The latter reference also gave the closed-form solution for the solution-existence condition. In addition, Zhang and Zhou[13] studied the tangent orbit technique in 3D based on a new definition of orbit "tangency" condition for noncoplanar orbits. Different from the orbital transfer problem, the orbital rendezvous problem requires the same time-of-flight for both the chaser and the target. Zhang et al. solved the two-impulse rendezvous problem between two coplanar elliptic orbits with only the second impulse[14] and both impulses being tangent,[15] respectively. Furthermore, Zhang et al.[16] solved the two-impulse rendezvous problem between coplanar elliptic and hyperbolic orbits.

This paper studies the coplanar orbit escape problem from an elliptic orbit to an excess velocity vector only with the tangent impulse. For a given initial orbit and a given excess velocity vector, the one-tangent-impulse transfer trajectory is obtained by solving a single-variable piecewise function. Then the optimal multi-tangent-impulse escape trajectory is obtained by the genetic method.

2. Orbital elements of transfer orbit

Assume that the spacecraft moves in a given initial orbit, a tangent impulse $\Delta\boldsymbol{v}$ with magnitude λ is imposed at P_1 (see Fig. 1), where the position vector relative to the Earth's center F_1 is $\boldsymbol{r}_0$ and the velocity vector is $\boldsymbol{v}_0$, then the velocity vector of the transfer orbit (or the final hyperbolic orbit) at P_1 is

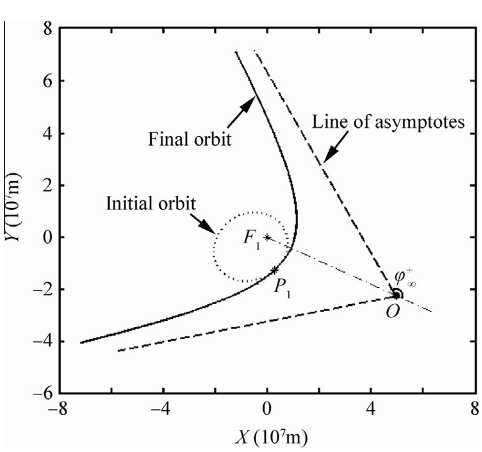

Fig. 1 Transfer to an excess velocity vector by a tangent impulse, example 1.

$$\boldsymbol{v}_\mathrm{f} = \boldsymbol{v}_0 + \Delta\boldsymbol{v} = (v_0 + \lambda)\boldsymbol{v}_0/v_0 \tag{1}$$

where $r_j = \|\boldsymbol{r}_j\|$, $v_j = \|\boldsymbol{v}_j\|$, $j = 0$, f, and 0 and f denote the initial and final orbits, respectively. A relationship between the magnitudes of velocity and position vectors is

$$v_j = \sqrt{\mu\left(\frac{2}{r_j} - \frac{1}{a_j}\right)} \tag{2}$$

where μ is the standard gravitational parameter, and a the semimajor axis. Then, the magnitude of the velocity can be written as a function of the initial true anomaly,

$$v_0 = \sqrt{\frac{\mu}{p_0}(1 + e_0^2 + 2e_0\cos\varphi_0)} \tag{3}$$

where φ is the true anomaly, φ_∞^+ the true anomaly at infinity of the excess hyperbolic orbit, e the eccentricity, and p the semilatus rectum.

From the energy equation, the semimajor axis of the final hyperbolic orbit is

$$a_\mathrm{f} = -\frac{\mu}{\left(v_\infty^+\right)^2} \tag{4}$$

where v_∞^+ is the excess velocity vector. Substituting Eq. (4) into Eq. (2) gives

$$v_\mathrm{f} = \sqrt{\left(v_\infty^+\right)^2 + \frac{2\mu}{r_0}} = \sqrt{\left(v_\infty^+\right)^2 + \frac{2\mu}{p_0}(1 + e_0\cos\varphi_0)} \tag{5}$$

Thus, for a given initial true anomaly φ_0 of the impulse point, the magnitude of the tangent impulse is

$$\begin{aligned}\lambda &= v_\mathrm{f} - v_0\\ &= \sqrt{\left(v_\infty^+\right)^2 + \frac{2\mu}{p_0}(1 + e_0\cos\varphi_0)}\\ &\quad - \sqrt{\frac{\mu}{p_0}(1 + e_0^2 + 2e_0\cos\varphi_0)}\end{aligned} \tag{6}$$

The eccentricity vector of the final orbit is

$$\boldsymbol{e}_\mathrm{f} = \frac{1}{\mu}\left[\left(v_\mathrm{f}^2 - \frac{\mu}{r_0}\right)\boldsymbol{r}_0 - (\boldsymbol{r}_0\cdot\boldsymbol{v}_\mathrm{f})\boldsymbol{v}_\mathrm{f}\right] \tag{7}$$

By using the following expression

$$\begin{aligned}\boldsymbol{r}_0\cdot\boldsymbol{v}_\mathrm{f} &= \left(1 + \frac{\lambda}{v_0}\right)(\boldsymbol{r}_0\cdot\boldsymbol{v}_0) = \left(1 + \frac{\lambda}{v_0}\right)\frac{\mu}{h_0}r_0 e_0\sin\varphi_0\\ &= \sqrt{\mu p_0}\left(1 + \frac{\lambda}{v_0}\right)\frac{e_0\sin\varphi_0}{1 + e_0\cos\varphi_0}\end{aligned} \tag{8}$$

the eccentricity vector in Eq. (7) can be written as

$$\begin{aligned}\boldsymbol{e}_\mathrm{f} = \frac{1}{\mu}\Bigg\{&\left[\left(v_\infty^+\right)^2 + \frac{\mu}{p_0}(1 + e_0\cos\varphi_0)\right]\boldsymbol{r}_0\\ &-\sqrt{\mu p_0}\left(1 + \frac{\lambda}{v_0}\right)^2\frac{e_0\sin\varphi_0}{1 + e_0\cos\varphi_0}\boldsymbol{v}_0\Bigg\}\end{aligned} \tag{9}$$

whose magnitude is the eccentricity e_f of the final orbit.

The normalized eccentricity vector is

$$\hat{\boldsymbol{e}}_\mathrm{f} = \frac{\boldsymbol{e}_\mathrm{f}}{e_\mathrm{f}} = \begin{bmatrix}\cos\omega_\mathrm{f}\cos\Omega_\mathrm{f} - \sin\omega_\mathrm{f}\sin\Omega_\mathrm{f}\cos I_\mathrm{f}\\ \cos\omega_\mathrm{f}\sin\Omega_\mathrm{f} + \sin\omega_\mathrm{f}\cos\Omega_\mathrm{f}\cos I_\mathrm{f}\\ \sin\omega_\mathrm{f}\sin I_\mathrm{f}\end{bmatrix} \tag{10}$$

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