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Achieving the capacity of half-duplex degraded relay channels using polar coding

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Abstract In this paper, a novel transmission protocol based on polar coding is proposed for the half-duplex degraded relay channel. In the proposed protocol, referred to as the partial message relaying, the relay only needs to forward a part of the decoded source message that the destination needs according to the exquisite nested structure of polar codes. Theoretically, it is proved that the scheme can achieve the capacity of the half-duplex relay channel under the decode-and-forward (DF) cooperation strategy while enjoying low encoding/decoding complexity. Practically, in order to minimize the global transmission power, the optimization of the power allocation is performed between the source and the relay by using information theoretic tools. Furthermore, a joint iterative soft parallel interference cancellation receiver structure is developed to suit to the proposed scheme. Simulation results show that the proposed scheme outperforms the conventional scheme designed by low-density parity-check (LDPC) codes.

1. Introduction

A widely used relay channel consists of three nodes,[1] of which the relay node helps to forward information from the source node to the destination node. In the past few years, there has been a significant progress in deriving the capacity of relay channels with different relay protocols.[2–7] In Ref.[8], the capacity of the half-duplex degraded relay channel is derived based on decode-and-forward (DF) cooperation strategy. A capacity-achieving scheme based on block Markov encoding is proposed but the decoding is based on joint typical sequences, which are of very high complexity. A practical scenario using low-density parity-check (LDPC) codes has been proposed in Ref.[9] to approach the capacity of the half-duplex relay channel with low decoding but high encoding complexity.

Very recently, polar codes have emerged as an efficient capacity-achieving coding scheme.[10] They were initially designed for a point to point binary-input discrete memory less channel (B-DMC) with low encoding and decoding complexity based on a phenomenon called channel polarization. At first, the theoretical bounds of polar codes were extensively studied in Refs.[11,12], while the practical performance of polar codes was improved significantly in Refs.[13–15] Later on, polar codes

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were used for certain multi-user schemes, such as multiple-access channels (MAC)[16,17] and relay channels.[18–20] Specifically, polar codes were designed for a 2-user MAC in Ref.[16] It has been shown that polar codes can achieve the capacity region of the 2-user MAC with similar complexity as the point to point case. In fact, the literature focusing on polar codes for the single relay channel encompasses, by now, only a few papers. In Ref.[18], polar codes were applied to constructing a capacity-achieving coding scheme for the physically degraded binary-input relay channel with orthogonal receiver. This work was further extended to arbitrary q-ary input in Ref.[19] Finally, without the assumption of orthogonally as in Ref.[20], polar codes by implementing block Markov coding were proved to achieve the capacity of the binary input symmetric physically degraded relay channel. However, all the work in Refs.[18–20] relies on the assumptions that the relay works under a full-duplex mode (i.e., the relay receives and transmits at the same time phases or using the same frequency bands) or the destination has orthogonal receiver components which avoid considering interference between the received signals (at the expense of increased use of spectrum). In practical communication systems, we prefer to consider the relay working under a time-division half-duplex mode and the destination receives signals in the same frequency band, since these settings can simplify the design of systems and exploit the channel resources efficiently.

In this paper, we will develop a low-cost transmission scheme, called partial message relaying, to achieve the capacity of the time-division half-duplex degraded relay channel proved in Ref.[8] based on the newly proposed polar codes. Specifically, in the first phase, the source broadcasts, and the relay and the destination receive. After decoding, the relay, instead of transmitting side information (additional parity bits) as in Ref.[9], forwards a part of the decoded source message that the destination needs in the second phase. In this phase, we consider two fundamental extremes where the source and the relay transmit completely correlated or independent messages similarly as in Ref.[9] It is proved that the proposed scheme under successive cancellation (SC) decoding[10] for this half-duplex relay channel inherits the block error probability bound of polar codes while keeping the low encoding/decoding complexity. In practical systems, we compute the optimum fraction of the global power by using information theoretic tools[21] which can maximize the achievable rate for the relay systems. We also develop a practical joint iterative soft parallel interference cancellation (JISPIC) receiver structure when the destination receives completely independent codewords. In this paper, we mainly consider the additive white Gaussian noise (AWGN) channels with binary phase-shift keying (BPSK) modulation. Compared with the conventional LDPC schemes for the half-duplex relay system, we show via simulations that the performance obtained exceeds about 0.1 dB.

The rest of the paper is organized as follows. In Section 2, we introduce briefly the system model and the background of polar codes. We explain in detail the transmission schemes using polar coding for the two time phases in Section 3. Section 4 gives the main result of this paper and establishes the proof of the theorem. The practical scheme for the half-duplex relay channel and the receiver structure are described in Section 5. Section 6 evaluates the performance of our proposed scheme for finite block length by simulations. Section 7 concludes the paper.

2. Preliminaries

2.1. System model

Consider a binary-input discrete memoryless relay channel consisting of one source (S), one relay (R) and one destination (D) which is shown in Fig. 1. We assume a time-division half-duplex mode. In each normalized block, the overall transmission is divided into two phases, a broadcast (BC) phase and a multiple access (MAC) phase, with durations t and $1-t$, respectively. Specifically, in the BC phase, S broadcasts signal X_0 and the corresponding observations at R and D are denoted by Y_1 and Y_0, respectively. Upon receiving Y_1 from the source, the relay generates new signal X_1. In the MAC phase, the relay transmits X_1 and at the same time the source transmits signal X_2.The corresponding channel output at D is denoted by Y_2.

According to the landmark work on the general relay channel,[2] the upper bound C^{U} and the lower bound C^{L} on the capacity of the time-division half-duplex relay channels are derived from a new cut-set theorem and a novel coding scheme,[8] which are given, respectively, by

$$C^{\mathrm{U}} \leqslant \sup_{0\leqslant t\leqslant 1} \min\{tI(X_0;Y_0,Y_1)+(1-t)I(X_2;Y_2|X_1),tI(X_0;Y_0)+(1-t)I(X_1,X_2;Y_2)\} \tag{1}$$

$$C^{\mathrm{L}} \geqslant \sup_{0\leqslant t\leqslant 1} \min\{tI(X_0;Y_1)+(1-t)I(X_2;Y_2|X_1),tI(X_0;Y_0)+(1-t)I(X_1,X_2;Y_2)\} \tag{2}$$

where $I(X; Y)$ denotes the mutual information for the channel with a binary input X and the corresponding output Y. The above mutual information expressions can be computed for AWGN channels with binary inputs by numerical integration and they can be easily evaluated by using Monte-Carlo simulations.[9]

If such a relay channel is considered to be physically degraded in the sense that

$$p(y_0',y_1',y_2'|x_0',x_1',x_2') = p(y_1'|x_0')p(y_0'|y_1')p(y_2'|x_1',x_2') \tag{3}$$

where the realizations of random variables $x_0' \in X_0, x_1' \in X_1, x_2' \in X_2, y_0' \in Y_0, y_1' \in Y_1, y_2' \in Y_2$, then the achievable lower bound is in fact the corresponding capacity of the half-duplex degraded relay channel which is given by

$$C = \sup_{t,p(x_0)p(x_1,x_2)} \min\{tI(X_0;Y_1)+(1-t)I(X_2;Y_2|X_1),tI(X_0;Y_0)+(1-t)I(X_1,X_2;Y_2)\} \tag{4}$$

As proved in Ref.[8], this capacity is achieved when all x_0', x_1' and x_2' are uniformly distributed and the time allocation factor t $(0 < t < 1)$ is optimized such that the two terms in the

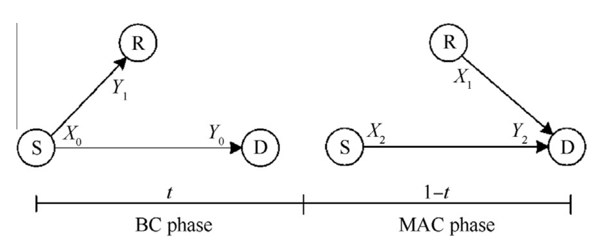

Fig. 1 Diagram of time-division half-duplex single-relay channel.