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Chance, determinism and the classical theory of probability

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ABSTRACT

This paper situates the metaphysical antinomy between chance and determinism in the historical context of some of the earliest developments in the mathematical theory of probability. Since Hacking's seminal work on the subject, it has been a widely held view that the classical theorists of probability were guilty of an unwitting equivocation between a subjective, or epistemic, interpretation of probability, on the one hand, and an objective, or statistical, interpretation, on the other. While there is some truth to this account, I argue that the tension at the heart of the classical theory of probability is not best understood in terms of the duality between subjective and objective interpretations of probability. Rather, the apparent paradox of chance and determinism, when viewed through the lens of the classical theory of probability, manifests itself in a much deeper ambivalence on the part of the classical probabilists as to the rational commensurability of causal and probabilistic reasoning.

1. Introduction

One of the oldest and most enduring metaphysical antinomies in the history of philosophy is that which exists between chance and determinism. How, on the one hand, can we admit the possibility that we live in a deterministic world in which every event has a prior necessitating cause and, at the same time, acknowledge the manifest fact that certain events happen by chance? The aim of this paper is to situate this metaphysical question in the historical context of some of the earliest developments in the mathematical theory of probability.

The mathematical rudiments of probability were laid in the second half of the seventeenth and eighteenth centuries, making the theory of probability, in its classical form, a product of Enlightenment thought.[1] Accordingly, most of the classical theorists of probability, in keeping with the rational optimism of their day, were self-avowed determinists, accepting that the explanatory achievements of Kepler and Newton in the domain of celestial mechanics could in principle be extended so as to encompass all natural phenomena.[2] There is no clearer statement of this unqualified determinism than that which appears in the following passage from Laplace:

> We ought then to consider the present state of the universe as the effect of its previous state and as the cause of that which is to follow. An intelligence that, at a given instant, could comprehend all the forces by which nature is animated and the respective situation of the beings that make it up, if moreover it were vast enough to submit these data to analysis, would encompass in the same formula the movements of the greatest bodies of the universe and those of the lightest atom. For such an intelligence nothing would be uncertain, and the future, like the past, would be open to its eyes. (Laplace, *Essai philosophique sur les Probabilités*, L 7 vi-vii)

This now famous depiction of a causally deterministic universe is part of a more extended panegyric in which Laplace heaps praise upon the sciences for having, at last, revealed the appearance of chance in nature to be an illusion. It is the progress of science towards the ultimate aim of expelling the 'blind chance of the

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[1] In what follows, the 'classical theory of probability' signifies the assortment of historical approaches to the subject which place an emphasis on combinatorial methods of reasoning, or accept, in some form, the classical definition of probability as the ratio of favorable to total possible outcomes, all such outcomes being equally possible. In accordance with this usage, classical theorists of probability include Pascal, Fermat, Huygens, Leibniz, Jacob Bernoulli, Montmort, de Moivre, and Laplace. The historical developments recounted by Todhunter (1865) and Hald (1990), with the exception of the latter's exclusion of Laplace, can be roughly identified with the history of the classical theory of probability.

[2] For representative statements of determinism in the writings of the classical theorists of probability, see B 239; Montmort (1713, xiv); and de Moivre (1756, 253). For more on the determinism of the classical theorists of probability, see Hacking (1990, ch.2); Daston (1995, 34–37); and Gigerenzer et al. (1990, 11–13).

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Epicureans' from the natural world which, for Laplace, 'distinguishes nations and ages, and constitutes their real glory.'[3]

Such rationalistic sentiments, while perhaps expressed in hyperbolic terms, are entirely in keeping with the intellectual orthodoxy of Laplace's day.[4] What gives to them an air of paradox, however, is the particular textual context in which they are made. For the above passage appears in the introduction to Laplace's *Essai philosophique sur les Probabilités*. Thus, following his unqualified rejection of even the slightest indeterminacy in the natural causes of things, Laplace proceeds to demonstrate, by means of a protracted series of calculations, how to compute with numerical exactitude the probabilities of various chance events. No justification is offered for this stark juxtaposition in the text other than the remark, made almost in passing, that 'probability is relative in part to [our] ignorance and in part to our knowledge.'[5]

Laplace and the other classical theorists of probability thus personified, in a particularly striking way, the antinomy between chance and determinism. On the one hand, in their capacity as Enlightenment thinkers they regarded chance as a fiction, 'a mere word,' its apparent reality reflecting only our incapacity to see all the way through to the underlying causes of things.[6] At the same time, in laying the groundwork for the mathematical theory of probability, they were busy conducting the first ever systematic investigation into the mathematical structure of chance phenomena. In light of this obvious tension it might seem inevitable that the classical theorists of probability would have made some effort to clarify the subject matter of their new 'géométrie du hasard.' Yet the intuitiveness of the methods they employed and the manifest practical value of the results thus obtained rendered otiose such philosophical subtleties. On the few occasions when such questions were addressed, the classical probabilists tended to prevaricate, offering a few brief qualifications to avoid any overt contradiction before returning to their calculations.[7]

This lack of clarity has led to the charge of incoherence in the classical conception of probability. Since Hacking's seminal work on the subject this charge has most often taken the form of an accusation that the classical probabilists were guilty of an unwitting equivocation between a subjective, or epistemic, interpretation of probability, on the one hand, and an objective, or statistical, interpretation, on the other.[8] Thus, for example, Gigerenzer et al. write:

> The classical interpretation of mathematical probability was characterized in precept by determinism and therefore by a subjective slant, and in practice by a fluid sense of probability that conflated subjective belief and objective frequencies with the help of associationist psychology. (Gigerenzer et al., 1990, 13)

Admittedly, there is some merit to this charge, for there does reside a fundamental conceptual tension at the heart of the classical theory of probability. As I hope to show, however, this tension is not best understood in terms of the contrast between subjective and objective interpretations of probability. Rather, the apparent antinomy of chance and determinism reflects a much deeper ambivalence, on the part of the classical probabilists, as to the commensurability of causal and probabilistic reasoning. This ambivalence is most clearly on display in the seemingly contradictory appeals to the principle of sufficient reason that appear in their writings. The challenge of developing a satisfactory account of the structure of rationality that can reconcile such diverse applications of this principle represents the true philosophical legacy of the theory of probability in its classical form.

The plan of the paper is as follows. In Section 2, I provide a brief introduction to the classical theory of probability, emphasizing the role that simple games of chance played in its early development. In Section 3, I examine the grounds of the judgments of 'equipossibility' underlying the combinatorial methods employed by the classical probabilists in the analysis of such games, and show that, ultimately, these judgments rely on an appeal to the principle of sufficient reason. Since it was this very same principle that led the classical theorists of probability to adopt a causally deterministic metaphysics, the antimony between chance and determinism, in this context, takes the form of a paradox of sufficient reason. In Section 4, I argue that the assumptions that are needed in order to generate the paradox cannot be reconciled with the intended applications of the theory. To illustrate this point, I consider Roberval's objection to Fermat's combinatorial solution to the problem of points. In Section 5, I offer some brief, concluding remarks on the philosophical challenges raised by the classical theory of probability.

2. Games of chance and the classical theory of probability

On several occasions, Leibniz remarked that logic should concern itself with the study of probability, proposing that the most natural place to begin such an inquiry is with a detailed analysis of games of chance:

> I have more than once said that we should have a new kind of logic which would treat of degrees of probability … Anyone wanting to deal with this question would do well to pursue the investigation of games of chance … carefully reasoned and with full particulars. This would be of great value … since the human mind appears to better advantage in games than in more serious pursuits. (Leibniz, *Noveaux Essais*, A VI.6 466)

The prudence of Leibniz's advice is confirmed by the fact that it was through the analysis of simple games of chance, such as those which involve the tossing of coins, the casting of dice, or the drawing of cards from a shuffled deck, that the theory of probability underwent its first growth in the direction of a mature and mathematically rigorous science.[9] It is worth considering briefly why this should have been the case. That is, why should so specialized an activity as dice casting have served as the backdrop for

[3] L 7 vii.

[4] As van Strien (2014, 27) reports, similar formulations of determinism in terms of an unbounded intelligence capable of predicting the future with certainty can be found in the works of several of Laplace's contemporaries, including Maupertius (1756, 332), Boscovich (1922, 281), Condorcet (1768, 5), and d'Holbach (1820, 51–52). Sheynin (1970, 234–6) regards Laplace's determinism as a development of the determinism of Newton and as a direct inheritance from Bernoulli and Condorcet (cf. Sheynin (1976, 172–4); Hahn (1967)). Hacking (1983) argues that the widespread belief in Laplacean determinism began to erode in the nineteenth century.

[5] L 7 viii.

[6] de Moivre (1756, 253).

[7] Bernoulli, for example, evades the issue of the apparent conflict between determinism and contingency as follows: 'In themselves and objectively, all things under the sun, which are, were, or will be, always have the highest certainty … Others may dispute how this certainty of future occurrences may coexist with the contingency … of secondary causes; we do not wish to deal with matters extraneous to our goal.' (B 239).

[8] See, e.g., Hacking (2006, ch.2); cf. Daston (1994, 332–3).

[9] This, of course, is not to deny that the concept of probability has a long history that predates its first mathematical treatment in the seventeenth century (see, e.g., Maistrov (2014) and Franklin (2015)).

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