Studies in History and Philosophy of Science 65-66 (2017) 87–98

Contents lists available at ScienceDirect

Studies in History and Philosophy of Science

journal homepage: www.elsevier.com/locate/shpsa

What's nu? A re-examination of Maxwell's 'ratio-of-units' argument, from the mechanical theory of the electromagnetic field to 'On the elementary relations between electrical measurements'

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ARTICLE INFO

Article history:
Received 3 April 2016
Available online 14 August 2017

Keywords:
James Clerk Maxwell
Electrical metrology
Electrodynamics
Absolute measurement
Units and dimensions
Electromagnetic theory of light

ABSTRACT

This re-examination of the earliest version of Maxwell's most important argument for the electromagnetic theory of light—the equality between the speed of wave propagation in the electromagnetic ether and the ratio of electrostatic to electromagnetic measures of electrical quantity—establishes unforeseen connections between Maxwell's theoretical electrical metrology and his mechanical theory of the electromagnetic field. Electrical metrology was not neutral with respect to field-theoretic versus action-at-a-distance conceptions of electro-magnetic interaction. Mutual accommodation between these conceptions was reached by Maxwell on the British Association for the Advancement of Science (BAAS) Committee on Electrical Standards by exploiting the measurement of the medium parameters—electric inductive capacity and magnetic permeability—on an arbitrary scale. While he always worked within this constraint in developing the 'ratio-of-units' argument mathematically, I maintain that Maxwell came to conceive of the ratio 'as a velocity' by treating the medium parameters as physical quantities that could be measured absolutely, which was only possible via the correspondences between electrical and mechanical quantities established in the mechanical theory. I thereby correct two closely-related misconceptions of the ratio-of-units argument—the counterintuitive but widespread notion that the ratio is naturally a speed, and the supposition that Maxwell either inferred or proved this from its dimensional formula.

1. Introduction

Maxwell's derivation of an equality between the speed of wave propagation in a hypothetical electromagnetic ether and the ratio of electrostatic to electromagnetic measures of electrical quantity was historically his most important argument for the electromagnetic theory of light.[1] From its first articulation in the early 1860s he sought more accurate measurements of the ratio, which he represented by the Greek letter ν, in order to demonstrate its identity with the speed of light. He argued that this provided strong grounds for believing that light was an electromagnetic wave and the optical and electromagnetic ether were one. Acceptance of this identity, Maxwell knew, substantiated his field-theoretic approach to electricity and magnetism at the expense of Continental action-at-a-distance theories.

A closer look reveals that Maxwell advanced a form of 'ratio-of-units' argument on multiple occasions: as part of the mechanical theory (1861–2) and the dynamical theory (1864) of the electromagnetic field; in a 'note on the electromagnetic theory of light' appended to his published measurements of ν (1868); and finally in the *Treatise on Electricity and Magnetism* (1873a). The earliest version of the argument, in Maxwell's mechanical theory, is unique among the others because it contained no suggestion that the ratio was either a speed or could be expressed as a speed.[2] It was only presented as such in a substantial piece entitled 'On the elementary relations between electrical measurements' (hereafter *OTER*), which Maxwell co-authored with the engineer Fleeming Jenkin as part of his work on the British Association for the Advancement of Science (BAAS) Committee on Electrical Standards (1863). This was also the first occasion where ν was used to designate the ratio and where dimensional considerations were drawn into the argument.

[1] For conceptual clarity, unless stated otherwise when I refer to 'the ratio' I will always have this definition in mind. Although Maxwell and others often talked about ν as the ratio of EM-to-ES *units*, or even the number of ES units in an EM unit, this was how he first defined it (see §4.3). The various definitions were treated as interchangeable (but see note 15 and Table 1 in §5 below).

[2] I follow Maxwell and his contemporaries by using 'speed' and 'velocity' interchangeably in relation to the ratio.

http://dx.doi.org/10.1016/j.shpsa.2016.08.005
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In short, a subtle yet substantive (and unacknowledged) conceptual shift had taken place.

This motivates a thorough reexamination of Maxwell's ratio-of-units argument from the mechanical theory to *OTER*, which reveals an unforeseen role for his model of the ether in his theoretical electrical metrology. Despite appearances to the contrary, absolute electrical metrology was not neutral with respect to field-theoretic versus action-at-a-distance conceptions of electro-magnetic interaction. Through Maxwell's example, I explain how mutual accommodation between these conceptions could only be reached by measuring the medium parameters—electric inductive capacity and magnetic permeability—on an arbitrary scale.[3] While Maxwell always worked within this constraint in developing the units argument mathematically, I maintain that he came to conceive of the ratio 'as a velocity' by treating the medium parameters as physical quantities that could be measured absolutely, which was only possible via the correspondences between electrical and mechanical quantities established in the mechanical theory. In the process, I correct two closely-related misconceptions of the ratio-of-units argument: the counterintuitive but widespread notion that the ratio is naturally a speed, and the supposition that Maxwell either inferred or proved this from its dimensional formula.

The paper is structured as follows. To make room to investigate how the ratio was somehow *made into a velocity* by Maxwell, §2 discusses Wilhelm Weber's mode of conceiving and representing it, if not as a velocity or in units of velocity. §3 re-examines the mechanical theory from the perspective of the medium parameters and the ratio, which, through derived field-theoretic forms of Coulomb's fundamental force laws, framed Maxwell's theoretical electrical metrology. §4 consequently explains why he regarded his definitions of absolute electrical units for the BAAS Committee on Electrical Standards as provisional, and how this related to the specification of air as a standard medium. It then describes how Maxwell articulated a connection between the ratio and a speed, and defined the symbol 'v' for the first time, in the context of unit conversion and the derivation of dimensional formulae for electrical (and magnetic) units.

§5 constitutes the capstone of the paper by revealing the full impact of Maxwell's mechanical theory upon his theoretical electrical metrology. It draws upon the findings of the previous sections to advance a two-tiered argument against the assumption that Maxwell inferred that the ratio was a speed from its dimensional formula LT^{-1}. The first tier shows minimally that, in the more general case where the medium is left unspecified, Maxwell reasoned in the just opposite sense, namely from a speed to the dimensional formula. The second, more speculative tier attempts to determine the pattern of that reasoning. To complete the circle, §5 concludes by establishing that Maxwell re-expressed Weber and Kohlrausch's measurement of the ratio as a speed. Since Maxwell did not make a sophisticated attempt to unpack this notion or the symbol 'v' until his *Treatise on Electricity and Magnetism*, I largely postpone this interpretive task for another paper.

2. Weber's forms of expression for the ratios of electrical units

Weber's attempts to unify electrostatics, electrodynamics, and electromagnetic induction in a single fundamental law provided an essential touchstone for all that followed, whether they were based upon action-at-a-distance or, like Maxwell's, ascribed agency to an intervening medium. Weber's contributions to electrical metrology, both theoretical and experimental, were integral to his electrodynamics and similarly influential. Following Gauss's lead in magnetism, he defined a range of absolute electrical units that were subsequently extended, systematized, and disseminated by the BAAS Committee on Electrical Standards, in part by Maxwell himself (Jungnickel & McCormmach, 1986, pp. 70–7, 129–47; Darrigol, 2000, pp. 49–66).

Das *mechanische Maass der Stromintensität* verhält sich also

zum *magnetischen* wie $1:155370\,.\,10^6$,
zum *elektrodynamischen* wie $1:109860\,.\,10^6\ (=1:155370\,.\,10^6\,.\,\sqrt{\tfrac{1}{2}})$,
zum *elektrolytischen* wie $1:\ 16573\,.\,10^9\ (=1:155370\,.\,10^6\,.\,106\tfrac{2}{3})$.

Fig. 1. Weber and Kohlrausch (1893 [1856]), p. 604, represented the relative sizes of various absolute units of current intensity as proportions. Weber's 'mechanical' unit of current was twice as large as the electrostatic unit because it consisted of two electrostatic units of electrical quantity, one positive and one negative, flowing through the cross-section of a wire in unit time. His 'magnetic' unit is Maxwell and the BAAS Committee's electromagnetic unit (Assis et al., 2004, p. 24).

At the intersection of Weber's theoretical electrodynamics and metrology lay his joint experimental determination with Rudolph Kohlrausch of the numerical relation between electrostatic and electromagnetic units of electrical current. This determination served a dual purpose for Weber. It would provide a value for c, the constant that featured in his fundamental force law, and enable a numerical comparison between the various absolute electrical units that he had previously defined (Assis, Reich, & Wiederkehr, 2004, pp. 17–28).

Since Weber already knew all their sizes relative to the electromagnetic unit, a single experiment would suffice. It involved comparing the measure, q, of electrical quantity stored by a Leyden jar in electrostatic units with an equivalent amount in electromagnetic units transmitted by an electrical current (for the definitions of these units, see §4.2).[4] These were determined by measuring the magnitude of the initial deflection of an oscillating magnetic needle in a galvanometer when the jar was discharged, and calculating the time τ during which a constant current of unit intensity in electromagnetic units had to flow to produce an equivalent deflection.[5] The ratio of electrostatic to electromagnetic measures of electrical current was given by q divided by τ (Weber & Kohlrausch, 2003 [1856], pp. 290–4; Darrigol, 2000, p. 66; Assis et al., 2004, p. 27).

Maxwell's appropriation of Weber and Kohlrausch's data in support of his electromagnetic theory of light has proven so successful that historians have frequently appended units of millimetres-per-second to their values (Siegel, 1991, pp. 122–3; D'Agostino, 2000, p. 31; Hunt, 2010, pp. 103, 106). But Weber and Kohlrausch never did so. They had indeed adopted millimetres and seconds as base units of length and time, but referred explicitly to the ratio of current units as a *number*. All other ratios of units were subsequently written *as ratios*, that is to say, in the standard notation for proportionality (see Fig. 1). Weber knew that these proportions varied directly with the unit of length and inversely with the unit of time—in modern terms, were homogeneous with the (absolute) unit of velocity—but did not convey this property in the

[3] Maxwell tended to use the term 'medium' to refer interchangeably to a given physical substance and the ether within it, whose properties he conceived as modifiable by that substance. Since it is usually clear which one he was referring to, I reserve the term 'medium' for the substance and distinguish this from ether-talk. This aids conceptual clarity except in cases where 'medium' in the sense of 'ether' appears frequently in quotations. This difficulty is especially apparent in §5.2.

[4] Weber and Kohlrausch used the symbol E instead of q; I have switched to q in order to avoid confusion with Maxwell's E (see §3).

[5] Only the initial deflection mattered because the discharge time was short compared to the oscillation period of the needle. Weber and Kohlrausch (2003) [1856] is an English translation of their (1893) [1856].