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What conceptual spaces can do for Carnap's late inductive logic

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A B S T R A C T

In the last published account of his late inductive logic, the Basic System of Inductive Logic, Rudolf Carnap introduced a new element to the systems of inductive logic, namely the so-called attribute spaces. These geometrical structures model the meanings of the predicates of the object language and have a similar structure as the conceptual spaces employed by cognitive scientists like Peter Gärdenfors. I show how the development of the theory of conceptual spaces helps us to see the addition of attribute spaces as a step forward in explicating the concept of confirmation. I discuss the differences and similarities of the two theories and investigate the possibilities for developing further connections.

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1. Introduction

In recent years we can observe two seemingly unrelated phenomena. The first one, related to the growing interest in formal philosophy, is that the interest in Rudolf Carnap's work is picking up even more momentum (Awodey & Klein, 2004; Carus, 2007; Leitgeb, 2011). The second is the emergence and strengthening of the theory of conceptual spaces, advanced mainly in cognitive science and related fields by authors such as Peter Gärdenfors. Conceptual spaces have been by now successfully applied also to classical philosophical problems (Zenker & Gärdenfors, 2015) and thus formal philosophers are beginning to take interest in them.

What is relatively unknown is that Carnap in his late work on inductive logic introduced and made heavy use of a notion very similar to today's conceptual spaces—namely, attribute spaces. The addition of the attribute spaces to inductive logic has not been discussed a lot in the literature since Hilpinen (1973), and the possible connections with the current work done in cognitive science have not received due attention (Gärdenfors (2004) mentions Carnap's work only in passing). This paper lies the foundations for developing connections between the two theories. In particular, I show how the theory of conceptual spaces can be seen as a factor in the explicative success of inductive logic.

When I mention Carnap's late work on inductive logic, I am referring to "A Basic System of Inductive Logic" (henceforth *the Basic System*), a two-part paper published as (Carnap, 1971a, 1980). Carnap concluded "Logical Foundations of Probability" (1950) with an exposition of the contents of the future second volume of that book—that was never published. In the course of the 1960s it became clear that his plan was not going to be realized and that the scope of another volume would have to be different. The Basic System is the result of those years' work and the system presented there was supposed to serve as a foundation for future inductive logic (see "Introduction" in (Carnap & Jeffrey, 1971)).

One big difference between the Basic System and Carnap's previous inductive logic is the introduction of the aforementioned attribute spaces—geometric representations of concepts—as another element of linguistic frameworks. The idea that such geometric structures could be introduced to model meaning relations between predicates was not entirely novel at the time. Van Fraassen (1967) already introduced a logical space—essentially a set of points representing some (not necessarily all) possible objects (which Carnap would think of as objects of a particular

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investigation)—used to model intensions of predicates. In this setting predicates correspond to subsets of the set of points of the space and hence intensional relations such as synonymy or inclusion of meaning can be read off directly from the space, avoiding the need for meaning postulates. Stalnaker (1981) used similar structures to define properties without the use of possible worlds.

Section 2 introduces the main elements of the Basic System, focusing on the notions of a linguistic framework and a confirmation function. Attribute spaces are then presented separately in Section 3, with the discussion of their role in the system in Section 4. All of the above sets the scene for Section 5 in which the addition of the attribute spaces is evaluated using the criteria for evaluating explications. This leads to the discussion of the theory of conceptual spaces in Section 6, followed by Section 7 which clarifies the possible relations between inductive logic and the theory of conceptual spaces, pointing out to some open questions.

2. The Basic System of Inductive Logic

Linguistic frameworks are a recurring theme in Carnap's writing and much of his work can be seen as a gradual elaboration of this notion. In the most general way the frameworks can be described as different language systems adopted in order to be used as a basis for scientific investigations. The closest Carnap comes in the Basic System to defining a linguistic framework is the following: "a universe of objects and a system of descriptive concepts that characterize the objects" (Carnap, 1971a, 47). The objects that a framework was designed to talk about are thus included as a part of the framework itself.

In the Basic System a linguistic framework is no longer just a language in the sense of a vocabulary and a set of rules governing the formation of expressions; it consists of many additional elements, namely a geometrical system of concepts (possibly, though not necessarily, supplied with a language) to be used for studying and describing a particular set of individuals, together with that set of individuals, and a number of meta-constraints and rules that determine the admissible confirmation functions for propositions about those objects. Later on it will be made clear what is meant by a 'geometrical system of concepts'; for now it suffices to say that in a linguistic framework the concepts are represented geometrically, and that in virtue of this representation the relations between concepts are derived from topological and metric properties of such a geometrical structure. Linguistic frameworks are set up for the purpose of conducting scientific investigations. Carnap describes a paradigmatic case of such activity as follows: "The person X wishes to assign rational credence values to unknown propositions on the basis of the observations he has made" (1980, 106).

2.1. Language and semantics

Although Carnap puts some stress onto the idea that concepts in a framework do not need to belong to a language, it still aids the clarity of exposition to present the formal apparatus of the system using the notion of a formalized language whose predicates correspond to the relevant concepts. "In this article the signs, formulas, and sentences do not play an essential role (…) at some occasions a reference to linguistic entities is made in order to facilitate understanding" (1971a, 47). Hence the notion of a formal language is still present. In the Basic System Carnap works with the same kind of formal language we recognize from his earlier works on probability. $\mathscr{L}$ is a monadic predicate language with (usually infinitely) countably many individual constants ($a_1, a_2, \ldots$), and a standard logical vocabulary (connectives, identity, quantifiers, individual variables). The logic of $\mathscr{L}$ is classical, as is the metalogic of the system. The object language is chosen for an investigation and

A family of predicates	A modality
e.g. color names	e.g. color
⇓	⇓
Predicates	Attributes
R B G …	Red Blue Green …

Fig. 1. Predicates and modalities.

therefore the choice of its particular characteristics will depend on the nature of the investigation and its goals.

The basic predicates of the language represent primitive attributes (properties) of the objects: "observable properties of observable objects" (1971a, 43). They are ordered into finitely many families. Two properties belong to the same family if they belong to the same general kind—here called a modality—for instance a modality of color or age[1] (see Fig. 1). The families are disjoint and exhaustive, i.e. it is logically necessary for any object to have at least one of the attributes of the family and it is inconsistent for it to have more than one of these properties. The division of predicates into families was already mentioned in the Logical Foundations (1950, §18c), where Carnap speaks of *families of related attributes*. However, it wasn't until the Basic System that the modalities were introduced as the factor determining the families.

Modalities are vital for the structure of the framework, since the division of the language into families is governed by them. As to what they are, Carnap only says that modalities are "general kinds" to which predicates of the same family belong, and that they can be qualitative (e.g. color, shape, substance) or quantitative (e.g. age, height). The best way to think about modalities is as those respects in which objects can be judged similar or different. While the attributes subsumed under a modality are comparable (e.g. we can say that red and orange are more similar to each other than red and blue), the modalities themselves are not (e.g. it is not immediately meaningful to say that color is more similar to shape than color is to sound).

Models for the object language are interpretation functions assigning attributes (properties) to individuals from the chosen domain. The domain is assumed to be always countable, and usually infinite, and for every individual there is exactly one constant denoting it. The initial set of all models—that is, of all the possible ways of assigning predicates to the individuals—can be further restricted by meaning postulates assumed for a given investigation (different kinds of postulates are discussed in Section 5 and an example of the restriction procedure can be found on pp. 82-83 of (Carnap, 1971a)). As will be seen shortly, the set of models for a given investigation becomes the sample space over which conditional probability functions representing confirmation will be assigned.

Propositions are sets of models, that is events on the sample space for the confirmation measures. Atomic propositions on $\mathscr{L}$ correspond to atomic sentences of $\mathscr{L}$, which are of the form $P_j a_i$ (where a_i is an individual constant). Hence atomic propositions are sets of those models in which $P_j a_i$ holds. The class of all propositions, denoted $\mathscr{E}_{\mathscr{L}}$, is a σ-field on the set of all models, generated by the collection of atomic propositions (1971a, 37). The latter means that $\mathscr{E}_{\mathscr{L}}$ is closed under complementation and finite and countably infinite unions. $\mathscr{E}_{\mathscr{L}}$ is very rich: for each class of sentences of the language, there is a corresponding proposition in $\mathscr{E}_{\mathscr{L}}$ (1971a, 60).

[1] In modern linguistics such families of predicates are referred to as domains, see (Langacker, 1987, chap. 4).