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The implementation, interpretation, and justification of likelihoods in cosmology

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ABSTRACT

I discuss the formal implementation, interpretation, and justification of likelihood attributions in cosmology. I show that likelihood arguments in cosmology suffer from significant conceptual and formal problems that undermine their applicability in this context.

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1. Introduction

In recent decades cosmologists have increasingly made use of arguments that involve the assignment of probabilities to cosmological models, usually as a way of guiding further theorizing about the universe. This despite cosmology being, on the face of it, an unlikely subject in which to employ probabilistic reasoning. In usual applications the utility of probabilities depends on their connection to empirical frequencies. In cosmology there is, so far as we know, only one universe. It would therefore seem to be an almost pointless exercise to attribute probabilities to the universe, its particular creation, or its particular history, as the assignment of probabilities would apparently be completely arbitrary. Nevertheless, perhaps owing to the significant observational limitations that exist in cosmology, cosmologists have sought to bolster the available empirical evidence with probabilistic reasoning, maintaining that it is both important and sensible to do so.[1]

Not only is strictly probabilistic reasoning salient in cosmology, but so are various other arguments which are similar in style to probabilistic reasoning. I will refer to such reasoning in general as *likelihood* reasoning. For example, typicality and some topology-based arguments do not rely on probabilities per se, but, like many probabilistic arguments, they aim to show that some conclusion or kind of outcome is, for example, typical or atypical, probable or improbable, or favored or disfavored, i.e. *likely* or *unlikely*.[2]

While the logical structure of such arguments is similar, the formal implementation, interpretation, and justification of the likelihoods themselves can differ significantly. The aim of this paper is to investigate these three features of likelihoods in order to determine the applicability of likelihood reasoning in cosmology. Although it is not possible to show that such reasoning definitively fails in all cases, I will argue that the various challenges I discuss do significantly undermine its viability in this context. These challenges include both conceptual and formal issues.

Before turning to these issues, however, it is appropriate to say a little more about the kind of arguments with which I am concerned. In an influential paper, Gibbons, Hawkings, and Stewart

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[1] It is not hard to find cosmologists expressing the importance of such arguments in cosmology: "The problem of constructing sensible measures on the space of solutions is of undeniable importance to the evaluation of various cosmological scenarios" (Gibbons & Turok, 2008, 1); "…the measure could play an important role in deciding what are the real cosmological problems which can then be concentrated on. In other words, we assume that our Universe is typical, and only if this was contradicted by the experimental data would we look for further explanations" (Coule, 1995, 455-6); "Some of the most fundamental issues in cosmology concern the state of the universe at its earliest moments, for which we have very little direct observational evidence. In this situation, it is natural to attempt to make probabilistic arguments to assess the plausibility of various possible scenarios (Schiffrin & Wald, 2012, 1).

[2] A referee notes the common technical usage of the term "likelihood" in statistics or more broadly in Bayesianism. I do not mean it in any technical sense but rather as a general term that covers kinds of reasoning similar to probabilistic reasoning.

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(GHS) give a concise formula for how likelihood reasoning is applied in cosmology:

> Cosmologists often want to make such statements as "almost all cosmological models of a certain type have sufficient inflation," or "amongst all models with sufficient baryon excess only a small proportion have sufficient fluctuations to make galaxies." Indeed one popular way of explaining cosmological observations is to exhibit a wide class of models in which that sort of observation is "generic." Conversely, observations which are not generic are felt to require some special explanation, a reason why the required initial conditions were favoured over some other set of initial conditions."(Gibbons et al., 1987)

GHS here suggest how such arguments can be used to guide further theorizing in cosmology. As they explicitly say, if some observed feature of the universe can be shown to be likely among the physically reasonable cosmologies, then it requires no further explanation; if it is unlikely, then it requires further explanation. Another variant goes as follows: if some unobservable feature of the universe is shown to be likely among physically reasonable cosmologies, then one infers that it exists; if it is unlikely, then one infers that it does not. In the following section I will provide an important example, fine-tuning of the standard model of cosmology, that follows these formulas.

I emphasize that there exist various formal implementations of likelihood that can be used to support this kind of argument, e.g. using topology, measure theory, probability theory, etc. Cosmologists, however, have generally favored those that are similar to the application of likelihoods in statistical mechanics, a context where likelihood reasoning is acknowledged as successful. Simply inferring from the success of arguments in statistical mechanics to similar ones in cosmology presupposes, however, that the justification and interpretation of likelihoods in statistical mechanics appropriately carries over to the cosmological context. I will argue that this presupposition is incorrect. Indeed, a central claim defended in this paper is that the justification and interpretation of cosmological likelihoods cannot be secured by similar strategies used to justify and interpret the use of likelihoods in statistical mechanics. I draw attention to this particular strategy at the outset because many cosmologists appear to take the problematic inferences for granted, and it is important to see that it is not viable. This is not the only strategy, of course, so its failure does not completely undermine likelihood reasoning in cosmology. Hence, although there is an emphasis on this particular strategy in the paper, in the main it concerns general challenges to implementing, interpreting, and justifying likelihoods in cosmology.

Although investigating the full complement of formal implementations of likelihood notions would be of interest, for reasons of simplicity, familiarity, and relevance to arguments made in the literature, I will concentrate mostly on probabilistic measures of likelihood. Although I will usually not generalize the considerations raised in the following to other formal implementations of likelihood, many of them do so generalize; the reader is therefore invited to keep these other implementations in mind. Nonetheless, at times I do consider topology- and typicality-based arguments explicitly.

Concerning probabilistic likelihoods specifically, recall that an application of probability theory standardly requires three things: a set X of possible outcomes (the "sample space"), a σ-algebra $\mathcal{F}$ of these possible outcomes (a collection of subsets that is closed under countable set-theoretic operations), and a probability measure P that assigns probabilities to elements of $\mathcal{F}$.[3] The probability spaces relevant for likelihood reasoning are those whose possible outcomes are possible cosmologies (models of the universe). Since the success of probabilistic arguments depends on an adequate justification of the relevant probability space and an adequate interpretation of probability in this context, I take as necessary conditions on a cosmological probability space that it be well-defined and that the choice of X and P must be justifiable and physically interpretable. (I take it that $\mathcal{F}$ can be chosen on essentially pragmatic grounds.) These are the implementation, interpretation, and justification conditions required for a probabilistic likelihood attribution. The challenges I raise in the following concern meeting these conditions.

The plan of the paper is as follows. I first provide (§2) a concrete example, fine-tuning problems with the standard model of cosmology, to further motivate and focus the subsequent investigation. In §3 I consider general conceptual issues of probability measures in cosmology, including the specification of the appropriate reference class X, and the interpretation and the justification of the probability measure P. The main conclusions of this section are that implementing cosmological probabilities can only be understood as an assignment of probabilities to initial conditions of the universe and, more importantly, that there is indeed no acceptable justification for any particular probability measure in the context of (single universe) cosmology. I then investigate the potential for formally implementing a measure associated with the space of possible cosmologies permitted by the general theory of relativity in §4. I point out a variety of significant obstacles to providing any such measure. One can avoid (or at least ignore) most of these general issues by truncating the spacetime degrees of freedom so that the relevant probability space is finite-dimensional. This is the approach taken to define the most discussed measure, the Gibbons-Hawking-Stewart (GHS) measure (Gibbons et al., 1987). In §5 I argue that even setting aside the problems raised in §§3-4 there are serious interpretive and technical problems with taking this narrower approach, in particular for supporting the fine-tuning arguments presented in §2. I offer concluding remarks in §6.

2. Fine-tuning problems in cosmology

To make the discussion more concrete, I will make use of a specific example involving likelihood arguments. Perhaps the most salient cases of likelihood reasoning in cosmology concern so-called "fine-tuning" problems.[4] Two of the most important fine-tuning problems in recent history are the hot big bang (HBB) model's flatness problem and horizon problem. They are important for my purposes because there is some reason to think that they are part of a successful chain of likelihood arguments, which I will briefly explain now.

The horizon and flatness problems begin with observations which suggest that the universe is, respectively, remarkably uniform at large scales and has a spatial geometry very close to flat. In the context of HBB model, the old standard model of cosmology, these presently observed conditions require very special initial

[3] Similarly, a topological space is specified by a set X (of possible spacetimes in this context) and a topology on X, i.e. a collection of subsets of X (the "open" sets). With a topology on X one can define a suitable notion of "negligible set" in the topology on X, for example a set whose closure has empty interior. The complements of negligible sets, "generic sets," are then sets with properties that are "almost always" possessed by the set X. In this way topology can be used to define a rough notion of likelihood: "almost always" and "almost never."

[4] Fine-tuning problems also appear elsewhere in physics. For example, in high energy physics the failure of naturalness in the standard model of particle physics, known as the hierarchy problem, is often described as a fine-tuning problem.

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