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Why did Einstein reject the November tensor in 1912–1913, only to come back to it in November 1915?

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A B S T R A C T

The question of Einstein's rejection of the November tensor is re-examined in light of conflicting answers by several historians. I discuss these conflicting conjectures in view of three questions that should inform our thinking: Why did Einstein reject the November tensor in 1912, only to come back to it in 1915? Why was it hard for Einstein to recognize that the November tensor is a natural generalization of Newton's law of gravitation? Why did it take him three years to realize that the November tensor is not incompatible with Newton's law? I first briefly describe Einstein's work in the Zurich Notebook. I then discuss a number of interpretive conjectures formulated by historians and what may be inferred from them. Finally, I offer a new combined conjecture that answers the above questions.

1. Introduction

Einstein detailed his struggles with mathematical tools that his loyal friend from school, Marcel Grossmann, brought him, in a small notebook from the winter of 1912–1913 – named by scholars the “Zurich Notebook”. Grossmann's name was written on top of one of the pages, where Einstein considered candidate field equations with a gravitational tensor constructed from the Ricci tensor; an equation Einstein would return to in his first November 1915 paper on general relativity. However, as indicated in the *Zurich Notebook*, he finally chose non-covariant field equations.

Without going into too many mathematical details here, I would like to present the basic problem. On page 19L Einstein constructed field equations out of the Ricci tensor in first-order approximation. These equations were written again on page 19R:

$$\Box g_{ik} = \kappa \rho_0 \frac{dx_i}{d\tau}\frac{dx_k}{d\tau} = \kappa T_{ik}. \quad (1)$$

The left-hand side of these weak-field equations included a reduced Ricci tensor, reduced to d'Alembertian operator in the weak-field limit:

$$\Box = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right),$$

while the right-hand side included the contravariant stress-energy tensor T_{ik} for a cloud of pressureless dust multiplied by the gravitational constant κ.

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On page 19R, Einstein introduced the linearized harmonic coordinate condition which was related with compatibility of the field equations and the Newtonian limit (Einstein, 1912a, pp. 433–437):

$$\sum_{\kappa}\gamma_{\kappa\kappa}\left(2\frac{\partial g_{i\kappa}}{\partial x_{\kappa}}-\frac{\partial g_{\kappa\kappa}}{\partial x_i}\right)=0. \tag{2}$$

The harmonic condition (2) was used to reduce the Ricci tensor to the d'Alembertian acting on the metric tensor field (metric), metric tensor, in the weak-field case, equation (1).

On page 19R, Einstein checked energy and momentum conservation for the gravitational field equation (1) in the case of weak-fields. However, he then realized that he needed an additional condition. He began by writing that for the first-order approximation our additional condition was obtained from the above harmonic coordinate condition. He suggested that the harmonic coordinate condition could perhaps be decomposed into two extra conditions. The first condition is the following:

$$\sum\gamma_{\kappa\kappa}\frac{\partial g_{i\kappa}}{\partial x_{\kappa}}=0\,. \tag{2a}$$

This condition is called the "Hertz condition" because it was later mentioned by Einstein in a letter to Paul Hertz (Einstein to Paul Hertz, August 22, 1915, *CPAE* 8, Doc. 111; Renn & Sauer, 2007, p. 184; Norton, 1984, p. 275).

Einstein imposed the (linearized) Hertz condition to ensure that his field equations (1) were compatible with energy-momentum conservation in first-order approximation. Consequently, the Hertz condition (2a) was added to make sure that the divergence of the stress-energy tensor vanishes in the weak-field case.

Einstein then realized that the combination of the harmonic coordinate condition and the Hertz condition caused a great problem because the trace of the weak-field metric is constant. Thus, the second condition, $\sum_{\kappa}\gamma_{\kappa\kappa}g_{\kappa\kappa}=const$, was a condition on the trace of the weak-field metric and it was incompatible with Einstein's conception of weak static gravitational fields: a spatial flat metric. Einstein's earlier work on static gravitational fields led him to conclude in the *Zurich Notebook* that in the weak-field approximation, the spatial metric of a static gravitational field must be flat (Einstein, 1912a, pp. 438–441).

On page 22R of the *Zurich Notebook*, perhaps at the suggestion of Marcel Grossmann, Einstein wrote the Ricci tensor in terms of the Christoffel symbols T_{il} and their derivatives. This way he obtained a fully covariant Ricci tensor in a form resulting from contraction of the Riemann tensor. Einstein divided the Ricci tensor into two parts – each of which separately transforms as a tensor under unimodular transformations – a tensor of second rank and a presumed gravitational tensor (Einstein, 1912a, pp. 451, 453). The presumed gravitational tensor was called by scholars the "November tensor":

$$T_{il}^{x}=\sum_{\kappa l}\frac{\partial}{\partial x_{\kappa}}\left\{\begin{matrix}il\\ \kappa\end{matrix}\right\}-\left\{\begin{matrix}i\kappa\\ \lambda\end{matrix}\right\}\left\{\begin{matrix}\lambda l\\ \kappa\end{matrix}\right\}. \tag{1a}$$

where $\left\{\begin{matrix}il\\ \kappa\end{matrix}\right\}$ are the Christoffel symbols of the second kind, see equation (5) below.

Setting the November tensor T_{il}^{x} equal to the stress-energy tensor, multiplied by the gravitational constant, one arrives at the field equations of Einstein's first paper of November 4, 1915.

Einstein no longer required the harmonic coordinate condition, and he could impose the Hertz coordinate condition to eliminate all unwanted second-order derivative terms. He hoped he could extract the Newtonian limit from the November tensor T_{il}^{x}. The Hertz condition also ensured that the divergence of the linearized stress-energy tensor vanished, thus satisfying energy-momentum conservation law. Einstein then imposed the Hertz condition on the first part of the Ricci tensor, the tensor of second rank, and was able to recover an expression that could lead him to Newton's law of gravitation:

$$-\frac{1}{2}\sum\gamma_{\kappa\alpha}\frac{\partial^2 g_{il}}{\partial x_{\alpha}\partial x_{\kappa}}-\frac{1}{2}\sum\left(\frac{\partial\gamma_{\kappa\alpha}}{\partial x_l}\frac{\partial g_{i\alpha}}{\partial x_{\kappa}}+\frac{\partial\gamma_{\kappa\alpha}}{\partial x_i}\frac{\partial g_{l\alpha}}{\partial x_{\kappa}}\right). \tag{3a}$$

He now imposed the Hertz condition on the second part of the Ricci tensor, on T_{il}^{x}, and recovered an expression that could lead him to Newton's law of gravitation:

$$\sum\frac{1}{2}\gamma_{\alpha\beta}\frac{\partial^2 g_{il}}{\partial x_{\alpha}\partial x_{\beta}}-\frac{1}{4}\sum\gamma_{\alpha\kappa}\gamma_{\beta\lambda}\left(\frac{\partial g_{i\alpha}}{\partial x_{\beta}}-\frac{\partial g_{i\beta}}{\partial x_{\alpha}}\right)\left(\frac{\partial g_{l\kappa}}{\partial x_{\lambda}}-\frac{\partial g_{l\lambda}}{\partial x_{\kappa}}\right)+ \tag{3b}$$

additional terms with products of first-order derivatives and Christoffel symbols of the first kind.

This expression eventually produces a Newton-Poisson equation as a first approximation.

On page 23L Einstein imposed a new coordinate condition to eliminate terms with unwanted second-order derivatives of the metric, and by which he hoped to extract the Newtonian limit from the November tensor T_{il}^{x}: the so-called "theta ϑ restriction" (see Section 2 for a detailed explanation on the ϑ restriction). He used the Hertz and ϑ conditions to eliminate various terms from T_{il}^{x}. The combination of the Hertz condition and the ϑ condition allowed Einstein to recover an expression from which the Newton-Poisson equation could again be obtained as a first approximation (Einstein, 1912a, pp. 454, 456):

$$\sum\gamma_{\kappa\alpha}\frac{\partial^2 g_{il}}{\partial x_{\kappa}\partial x_{\alpha}}+\sum\gamma_{\rho\alpha}\gamma_{k\beta}\frac{\partial g_{ik}}{\partial x_{\alpha}}\frac{\partial g_{l\rho}}{\partial x_{\beta}}. \tag{3c}$$

However, Einstein finally rejected both the truncated November tensor T_{il}^{x} and equations (3b) and (3c) (Einstein, 1912a, pp. 450–454, 456). He then abandoned the Hertz condition, and wrote that it was unnecessary.

Einstein already accumulated coordinate conditions (the harmonic condition, the Hertz condition, and the ϑ condition) to eliminate the terms from his equations, and he extracted expressions of broad covariance from the Ricci tensor. He then truncated them by imposing additional conditions on the metric to obtain candidates for the left-hand side of the field equations that reduce to the Newtonian limit in the case of weak static fields. However, this model entangled the Newtonian limit and conservation of momentum-energy.[1]

On pages 24R-25R, Einstein therefore tried to extract yet another candidate for the left-hand side of the field equations. He did not extract these field equations from the Ricci tensor but established field equations while starting from the requirement of the conservation of momentum and energy. He still hoped to connect the new field equations he found through energy-momentum considerations to the November tensor T_{il}^{x} of page 22R. However, he finally abandoned general covariance on the very next pages (26L and 26R) and derived the so-called *Entwurf* field equations – which he also established using the same method, through energy-momentum considerations. These equations could be covariant with respect to linear transformations, and they

[1] This topic will be further explained in Section 2 devoted to interpretation of the *Zurich Notebook* by a group of scholars.