Studies in History and Philosophy of Modern Physics xxx (2017) 1–16

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Klein-Weyl's program and the ontology of gauge and quantum systems

Gabriel Catren

Laboratoire SPHERE (UMR 7219), Université Paris Diderot - CNRS, 5 rue Thomas Mann, 75205 Paris Cedex 13, France

ARTICLE INFO

Article history:
Received 25 February 2016
Accepted 10 May 2017
Available online xxx

Keywords:
Hermann Weyl
Klein's Erlangen program
General relativity
Gauge theories
Quantum mechanics
Symmetries

ABSTRACT

We distinguish two orientations in Weyl's analysis of the fundamental role played by the notion of symmetry in physics, namely an orientation inspired by Klein's Erlangen program and a phenomenological-transcendental orientation. By privileging the former to the detriment of the latter, we sketch a group(oid)-theoretical program—that we call the *Klein-Weyl program*—for the interpretation of both gauge theories and quantum mechanics in a single conceptual framework. This program is based on Weyl's notion of a "*structure-endowed entity*" equipped with a "*group of automorphisms*". First, we analyze what Weyl calls the "*problem of relativity*" in the frameworks provided by special relativity, general relativity, and Yang-Mills theories. We argue that both general relativity and Yang-Mills theories can be understood in terms of a *localization* of Klein's Erlangen program: while the latter describes the *group-theoretical automorphisms of a single structure* (such as homogenous geometries), local gauge symmetries and the corresponding gauge fields (Ehresmann connections) can be naturally understood in terms of the *groupoid-theoretical isomorphisms in a family of identical structures*. Second, we argue that quantum mechanics can be understood in terms of a *linearization* of Klein's Erlangen program. This stance leads us to an interpretation of the fact that quantum numbers are "*indices characterizing representations of groups*" ((Weyl, 1931a), p.xxi) in terms of a correspondence between the ontological categories of *identity* and *determinateness*.

1. Introduction

H. Weyl was a principal actor in the two most important revolutions in the 20^{th} century physics, namely the geometrization of the fundamental interactions in the framework of the *gauge theories* on the one hand and *quantum mechanics* on the other. Regarding the former, Weyl gave the first steps to extend Einstein's geometrization of gravitation to the non-gravitational interactions. In particular, he provided—in the framework of his gauge theories of 1918 and 1929 (Weyl, 1918, 1929a)—the first formulation of the *gauge argument*, i.e. of the argument according to which the requirement of local gauge invariance dictates both the introduction of the gauge fields and the form of the interaction between the latter and the matter fields.[1] Weyl's first attempts finally led to the formulation of the *Yang-Mills theories* of non-gravitational fundamental interactions in 1954 (Yang & Mills, 1954). From a mathematical viewpoint, C. Ehresmann gave an important step in this history by unveiling the fundamental geometric structure underpinning Yang-Mills theories, namely the *Ehresmann connections* on principal fiber bundles (Ehresmann, 1950).

Regarding quantum mechanics, Weyl was one of the first—together with Wigner—to apply the theory of group representations to this theory (Weyl, 1927, 1931a) (see also Refs. (Mackey, 1980a; Mehra & Rechenberg, 2001; Scholz, 2006, 2007; Speiser, 1985) and Ref. (Mackey, 1993) for a comparison of Weyl's and Wigner's applications of the theory of group representations to quantum mechanics). What we could call—following Mackey (Mackey, 1980b)—*Weyl's program* amounts to understanding one

E-mail address: gabrielcatren@gmail.com.

[1] For a conceptual discussion of the gauge argument see Refs. (Brown, 1999; Catren, 2008a; Martin, 2002; Teller, 2000). For a discussion of the history of the gauge argument and a collection of the main articles in this history see Ref. (O'Raifeartaigh, 1995). It is worth noting that the term *gauge theory* has two interrelated meanings. First, it denotes Dirac's theory of constrained Hamiltonian systems (Dirac, 1964; Henneaux & Teitelboim, 1994). Second, it denotes the geometric description of the fundamental interactions in terms of connections on principal fiber bundles over spacetime. While the gauge theories of fundamental interactions have a constrained Hamiltonian formulation, there are Hamiltonian theories with constraints not arising from the description of connections on principal fiber bundles.

http://dx.doi.org/10.1016/j.shpsb.2017.05.003
1355-2198/

Please cite this article in press as: Catren, G., Klein-Weyl's program and the ontology of gauge and quantum systems, Studies in History and Philosophy of Modern Physics (2017), http://dx.doi.org/10.1016/j.shpsb.2017.05.003

of the most fundamental features of quantum mechanics—namely the commutation relations and the resulting Heisenberg indeterminacy principle—from a group-theoretical perspective. Weyl's attempt to ground quantum mechanics on a group-theoretical basis had—in spite of the negative reactions initially elicited against the *Gruppenpest*—a glorious posterity. Among the principal hallmarks in the history of the relations between quantum physics and group theory we can mention the group-theoretical classification of elementary particles (Souriau, 1997; Wigner, 1939), Mackey's systems of imprimitivity (Mackey, 1976; Varadarajan, 1985), Kirillov's orbit method (Kirillov, 2004), and the Kostant-Souriau geometric quantization formalism (Kostant, 1970; Souriau, 1997). Nonetheless, it is worth stressing that in spite of these ground-breaking formal achievements the group-theoretical foundational program launched by Weyl does not play (at least to the knowledge of the author) any *central* role in the leading interpretations of quantum mechanics (such as the Copenhagen interpretation, the hidden variables interpretations, the many-worlds interpretation, or the information-based interpretations).

Weyl's contributions to these "revolutions" in 20th century physics were oriented by two independent regulative leitmotifs.[2] On the one hand, Weyl's "*purely infinitesimal program*" (*Reine Infinitesimalgeometrie*), which underpinned his contributions to the development of gauge theory, was philosophically guided by what we shall generally call a *transcendental leitmotif*.[3] The influence of transcendental idealism in Weyl's thought, far from being homogenous, combines influences from both Husserl's transcendental phenomenology (see Refs. (Ryckman 2003a, 2005, 2009)) and the post-Kantian "constructivist" philosophy of Fichte (see Refs. (Scholz, 1995, 2005; Sieroka, 2007)).[4] Regarding the notion of symmetry, this transcendental leitmotif leads Weyl to the claim that the fundamental role played by symmetries in physics can be explained by means of *a priori* considerations regarding the transition—mediated by "*symbolic construction*"—from absolute subjective experience (of a qualitative *suchness* placed in an extended *Here-Now*) to relative objective knowledge.

On the other hand, Weyl's work and reflections on both the "*problem of relativity*" and quantum mechanics were also influenced by a *Kleinian leitmotif*. This regulative orientation results from the influence of both Klein's Erlangen program (Klein, 1872) (see also Ref. (Sharpe, 1997), Chap.4, and Ref. (Gray, 2005) for a historical discussion) and the development of group theory in the framework of crystallography.[5] Rather than addressing the *a priori* conditions of possibility of objective representation, this orientation is based on the (*a posteriori*) consideration of "*structure-endowed entities*" endowed with non-trivial *automorphisms* ((Weyl, 1952), p.144). In other terms, the analysis of the transcendental constitution of physical objectivity is substituted with the analysis of the intrinsic structures of particular (formal, physical, and cultural) objects, namely regular objects endowed with intrinsic symmetries (such as for instance Klein geometries, crystals, and ornaments). Weyl's book *Symmetry*—being a sort of naturalist compendium of regular structures found in mathematics, nature, and culture—can be understood as a paradigmatic expression of this "empirical" attitude regarding the notion of symmetry.

In what follows we propose a particular articulation of a certain number of ideas and statements extracted from Weyl's reflections on the notion of symmetry, namely

(1) that the *epistemic symmetries* related to the free election of a frame of reference are dependent upon the *intrinsic symmetries* resulting from the fact that the corresponding "*structure-endowed entity*" has non-trivial automorphisms (see for instance the Section III.13 "*The Problem of Relativity*" in Ref. (Weyl, 1949b)).
(2) that the knowledge of the group of automorphisms of a "*structure-endowed entity*" Σ provides a "*deep insight into the constitution of* Σ" ((Weyl, 1952), p.144),
(3) that the quantum numbers are "*indices characterizing representations of groups*" ((Weyl, 1931a), p.xxi),
(4) that "*objectivity means invariance with respect to the group of automorphisms*" ((Weyl, 1952), p.132).

We argue that the articulation of these statements points towards a *groupoid-theoretical comprehension* of both gauge and quantum systems based on the notion of a *structure-endowed entity* equipped with a *group of automorphisms*, i.e. with a group of transformations which leave the structure of the entity unchanged (see for instance (Weyl, 1952), p.42). To do so, we address the inner tension between the two aforementioned regulative orientations in Weyl's reflections by privileging a Klein-oriented interpretation of these statements to the detriment of the transcendental orientation. In order to stress this Kleinian inflection of what Mackey called *Weyl's program*, we call the resulting "program" for the interpretation of gauge and quantum systems *Klein-Weyl's program*. Transcendental arguments provide an extremely elegant and compelling explanation of the *fundamental* role played by the notion of *symmetry* in physics.[6] However, we argue that Klein's Erlangen program provides an alternative conceptual framework for understanding the fundamental role played by symmetries in

[2] In this article, we shall not analyze the possible chronological periodization of these two orientations. For an analysis of the evolution of Weyl's philosophical ideas see Refs. (Eckes, 2012; Scholz, 2005, 2011).

[3] Even if transcendental arguments do not play a central role in the framework of Weyl's works on quantum mechanics, he endorses the view that the "*central problem*" of this theory is an epistemic problem. In *The Theory of Groups and Quantum Mechanics*, Weyl explicitly uses a Kantian terminology to endorse a transcendental interpretation of quantum mechanics: "But scientists have long held the opinion that such *constructive concepts* [such as the Galilean concept of mass] *were nevertheless intrinsic attributes of the 'Ding an sich,'* even when the manipulations necessary for their determination were not carried out. *In quantum theory we are confronted with a fundamental limitation to this metaphysical standpoint*." ((Weyl, 1931a), p.76). In *Philosophy of Mathematics and Natural Science*, Weyl endorses an instrumentalist interpretation of the Heisenberg indeterminacy principle ((Weyl, 1949a), p.256, p.263). He concludes his summary of the features of quantum physics which seem to him of "paramount philosophical significance" by saying that "the meaning of quantum physics […] is not yet clarified […] The relation of reality and observation is the central problem. We seem to need a deeper epistemological analysis of what constitutes an experiment, a measurement, and what sort of language is used to communicate its result." ((Weyl, 1949a), p.264).

[4] It is worth noting the existence of a tension in Weyl's thought between the Husserlian "intuitionism" and the Fichtean "constructivism" (see Ref. (Sieroka, 2009) for an analysis of this point). However, we think that this tension can be sublated *within a transcendental framework* once we take into account that Kantian transcendental philosophy is essentially based on the articulation between what Kant calls *receptivity* and *spontaneity*.

[5] With respect to the influence of Klein's Erlangen program in Weyl's work on quantum mechanics see Ref. (Eckes, 2012). Regarding the historical importance of crystallography, Weyl writes in the introduction to *The Theory of Groups and Quantum Mechanics* that "*until the present, the most important application* [of the group concept] *to natural science lay in the description of the symmetries of crystals*." ((Weyl, 1931a), p.xxi).

[6] See notably the articles in the collected volume (Bitbol, Kerszberg, & Petitot, 2009). See also Refs. (Kauark-Leite, 2012; Ryckman, 2005) for an analysis of the role played by transcendental philosophy with respect to quantum mechanics and gauge theories respectively.

Please cite this article in press as: Catren, G., Klein-Weyl's program and the ontology of gauge and quantum systems, Studies in History and Philosophy of Modern Physics (2017), http://dx.doi.org/10.1016/j.shpsb.2017.05.003