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Metaphysics of the principle of least action

Vladislav Terekhovich

Institute of Philosophy, Saint-Petersburg State University, 5 Mendeleevskaya Liniya, 199034 St. Petersburg, Russia

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ABSTRACT

Despite the importance of the variational principles of physics, there have been relatively few attempts to consider them for a realistic framework. In addition to the old teleological question, this paper continues the recent discussion regarding the modal involvement of the principle of least action and its relations with the Humean view of the laws of nature. The reality of possible paths in the principle of least action is examined from the perspectives of the contemporary metaphysics of modality and Leibniz's concept of essences or possibles striving for existence. I elaborate a modal interpretation of the principle of least action that replaces a classical representation of a system's motion along a single history in the actual modality by simultaneous motions along an infinite set of all possible histories in the possible modality. This model is based on an intuition that deep ontological connections exist between the possible paths in the principle of least action and possible quantum histories in the Feynman path integral. I interpret the action as a physical measure of the essence of every possible history. Therefore only one actual history has the highest degree of the essence and minimal action. To address the issue of necessity, I assume that the principle of least action has a general physical necessity and lies between the laws of motion with a limited physical necessity and certain laws with a metaphysical necessity.

1. Introduction

The principle of least action (PLA) is one of the most general laws of theoretical physics and simultaneously one of the most philosophically controversial laws. Over the centuries, many scientists have linked it to hopes of a universal theory, despite the related metaphysical disputes about causality. Fermat, Leibniz, Maupertuis, and Euler were sure that nature is thrifty in all its actions thanks to the perfection of God. Planck believed that, "among the more or less general laws which manifest the achievements of physical science in the course of recent centuries, the Principle of Least Action is probably the one, which, as regards form and content, may claim to come nearest to that final ideal goal of theoretical research" (Stöltzner, 2003).

The PLA and other variational or extremal principles provide an alternative and more global approach to mechanics than Newton's laws. The PLA and the calculus of variation, in general, are more global than local differential equations and are widely used for solving dynamic tasks in diverse fields of physics such as classical mechanics, electrodynamics, relativity theory, and quantum physics. In the PLA, the action is the integral of a certain expression along a possible path or history of a system in a configuration space. The expression can be Lagrangian (for instance, the difference between kinetic and potential energy) or, in the case of continuous fields, the Lagrangian density. The integral can be over the path, time, n-dimensional volume, or four-dimensional space-time. In the quantum field theory, the action has the meaning of the phase of quantum amplitude (Feynman & Hibbs, 1965). In other variational principles, many characteristics take a minimal or maximal value from all possible values. These could include: the optical length, constraint, proper time, curvature of space-time, and thermodynamic potentials (Goldstein, Poole, & Safko, 2002; Hanc & Taylor, 2004; Lanczos, 1986; Landau & Lifshitz, 1975; Lemons, 1997; Ogborn & Taylor, 2005; Papastavridis, 2002; Sieniutycz & Farkas, 2005; Stöltzner, 1994, pp. 33–62; Taylor & Wheeler, 2000; Yourgrau & Mandelstam, 1968).

And yet, the PLA has always been surrounded by a fog of mysticism. The system seems to "choose" the actual path along which an action is less than along other paths. It is as if the system's final state determines the path that the system takes to reach that state. On the one hand, we cannot allege that an object actually "chooses" or "calculates" the path of minimal action. On the other hand, it appears that the actual path is somehow connected with the future actual state or event. A general principle of causality states that a cause should always precede its effect. This view of causality is used in most of the physical laws and is consistent with a reasonable belief that causal influences cannot travel backwards in time. Nevertheless, to the present day we have not understood

E-mail address: v.terekhovich@gmail.com.

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how a physical system seems to "choose" an actual path or history from all possibilities for motion or why this actual history involves minimal action. Moreover, the history of physical teleology might alternatively suggest a relationship between the PLA and the problem of determinism (Stöltzner, 2003). Besides sitting between teleology and determinism, the PLA takes a special place among other physical laws.[1] Additionally, it appeals to a modal notion of "possibilities".

Today, in spite of Planck's hope, the PLA is generally accepted only as a mathematical tool equivalent to the differential equations of motion (Yourgrau & Mandelstam, 1968, p. 178f). However, some physicists have tried to clarify the foundations of the variational principles (Polak, 1959; Asseev, 1977; Lanczos, 1986; Stöltzner, 1994, 2003; Yourgrau & Mandelstam, 1968; Wang, 2008). As Butterfield (2004a) stressed, the philosophical literature about variational principles has focused almost entirely on the specifying final conditions and referring to least action, suggests teleology. At the same time, philosophers have not explored the modal involvements of analytical mechanics. Butterfield noted that, thanks to the rise of modal metaphysics in analytical philosophy, the topic is plainly visible nowadays. Indeed, recently, some authors have examined how the PLA and other variational principles are involved in modal metaphysics (Bird, 2007; Butterfield, 2004a, 2004b; Ellis, 2005; Katzav, 2004, 2005; Terekhovich, 2013; Thébault & Smart, 2013). They have considered the relations between the PLA and causality, dispositional essentialism, the Humean view of the laws of nature, and the truthmaker principle. However, the study of modality is extensive and concerns some other issues connected with necessity and possibility. The themes of the modality and the nature of possible worlds are widely discussed in modal metaphysics (Adams, 1974; Armstrong, 2004; Chihara, 1998; Fine, 2005; Kripke, 1980; Lewis, 1986; Plantinga, 1974) and in relation to different physical phenomena (Bird, 2006; Ellis, 2001; Shoemaker, 1984).

This paper continues the recent discussion of the metaphysical issues of the PLA, especially regarding the modal involvement of the PLA. I think that Butterfield (2004a, 2004b) is right in that the whole of analytical mechanics is steeped in modality. I am, moreover, sure that the most promising direction for the PLA is the metaphysical investigation of the possible paths or histories connected with the laws of quantum systems. In addition to presenting criticism of other concepts, I propose a positive solution for the metaphysical content of the PLA. First of all, I examine the question of a reality of "possible paths" or "possible histories" in the PLA, as well as how they are connected with the notion of "possible objects" or "possibilia" of the contemporary metaphysics of modality and of Leibniz's concept of the essences or possibles striving for existence.

This paper's solution for some of the metaphysical issues of the PLA is based on the intuition that quantum mechanics might be a key to understanding the philosophical content of this principle. I assume that deep ontological connections exist between the possible paths of the PLA and quantum possible histories of the Feynman path integral formalism (FPI).[2] However, I do not reduce the metaphysical significance of the PLA to the quantum histories. The FPI is unlikely to have an independent metaphysical essence. Rather the ontological connections between PLA and FPI are made up in their possible histories, which obey the common metaphysical laws.

This paper introduces the model of a two-level modality based on a realistic approach to the possible or virtual motions in the calculus of variations. It considers the possible paths in the Feynman integral as being descriptions of similar processes taking place in the possible modality of being. I elaborate the modal interpretation of the PLA that replaces the classical representation of the system's motion along a single history in the actual modality by the simultaneous motions along the infinite set of all possible histories in the possible modality. To address the issue of necessity, I assume the PLA to have a general physical necessity and to lie between the laws of motion (with a limited physical necessity) and certain laws (with a metaphysical necessity) that govern the PLA. The rest of this paper is structured as follows. Section 2 gives a short description of the PLA. Section 3 illustrates the connection between the PLA and the FPI of quantum mechanics. Section 4 briefly introduces some metaphysical difficulties of the PLA related to causality, necessity, and the notion of possibility. Section 5 discusses the problem of the reality of the possible histories, possible objects, and possible worlds from the perspectives of various stances of modal metaphysics, including the Leibniz concept. The basic notions of the modal interpretation of the PLA are formulated in Section 6. The relations between the PLA and dispositional essentialism are considered in Section 7. Section 8 explains how the modal interpretation of the PLA can change the view of causality in the PLA. Section 9 compares the arguments in the debate regarding the Humean and non-Humean views of the laws of nature with concern to the PLA. Section 10 presents the paper's conclusions.

2. Principle of least action (PLA)

Let us consider two ways in which classical mechanics explains the motion of a falling apple: Newton's laws and Hamilton's principle of least action.[3]

2.1. Newton's laws of motion

Firstly Newton said: *Give me the apple's initial position and its velocity or two very nearby positions of the apple*. Then Newton answered the question: *What is the position of the apple at the next instant, if it is acted upon by Earth's gravity or some force?* Newton postulated the first law of motion or the principle of inertia. If there were no acting forces, the apple would possess a mysterious internal tendency to continue in motion with the same velocity along a straight line. The second law of motion postulated that a force such as Earth's gravity causes motion in the direction of the applied force. In other words, when the apple "perceives" at a distance the attraction of Earth[4] or the effect of the force, the apple is accelerated or changes its velocity. Thus, the path of the apple's actual motion is the result of the combining or summation of two tendencies or "effects": the apple's inertial motion and the motion due to the force acting on it. Finally, we obtain a differential equation to calculate all positions of the apple.

[1] As shown, the conservation laws can be derived from the variational principles (Goldstein et al., 2002; Hanc & Taylor, 2004; Brizard, 2008).

[2] It is known that the PLA is connected with the FPI through the notion of "action" and can be derived from the FPI as a limit on a large scale. Feynman even argued that the relation between symmetry laws and conservation laws is connected with the principle of least action "because they come from quantum mechanics" (Feynman, 1985, p. 105). Some authors have argued that all of classical mechanics could be represented as a short-wave approximation of quantum mechanics, and therefore, the action has the meaning of the phase of quantum amplitude (Feynman & Hibbs, 1965; Ogborn & Taylor, 2005; Taylor, 2003).

[3] In this section, I use the description of Feynman et al. (1964, pp. 19–2) and Hanc (2006). Hanc, moreover, explained three approaches: Newton's laws, Hamilton's and Maupertuis' principles of least action.

[4] As is well known, Newton did not insist on the univocal nature of gravity. For instance, he referred the motive force to the body "as an endeavor and propensity of the whole towards a centre" (Newton, 1962, p. 5).