Studies in History and Philosophy of Modern Physics 46 (2014) 164–169

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Three principles for canonical quantum gravity

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ARTICLE INFO

Article history:
Received 18 December 2012
Received in revised form
4 August 2013
Accepted 18 September 2013
Available online 25 October 2013

Keywords:
Loop quantum gravity
Diffeomorphism invariance
Local Lorentz invariance

ABSTRACT

We outline three principles that should guide us in the construction of a theory of canonical quantum gravity: (1) diffeomorphism invariance, (2) implementing the proper dynamics and related constraint algebra, (3) local Lorentz invariance. We illustrate each of them with its role in model calculations in loop quantum gravity.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

Since at present we do not have unexplained experimental evidence that requires a quantum theory of gravity for its understanding, we find ourselves in a rather unconventional situation. In physics, theory is usually guided by experiment. The situation is perhaps akin to the one faced by Einstein when developing the general theory of relativity. Although there were some experiments to be explained, he had to be mostly guided by physical principles and intuition. Here we would like to highlight three physical principles that we believe should provide guidance in canonical quantum gravity, and the implications of their use in some model situations.

The first principle is diffeomorphism invariance. No one believes that a fundamental theory of gravity should depend on background structures therefore space–time diffeomorphism invariance needs to be implemented. The history of how we ended up with background independence as a principle throughout the history of physics all the way back to the relational ideas of Mach is well recounted by Smolin. Modern gravity theories are, however, complicated. For instance in general relativity one has several layers of structure to consider. The most elementary is the dimensionality of the space–time. Then its topology. Furthermore there is the differential structure, the signature and finally the metric and fields. We will restrict our discussion to approaches that consider the dimension, differential structure and signature as given (although the introduction of certain measures in Hilbert spaces may imply a change in differential structure, one expects that in semiclassical regimes the differential structure is unchanged). Only diffeomorphism invariant questions about the metric and the fields can be considered physically relevant. Topology change can be accommodated in various approaches to quantum gravity, including the canonical one (Horowitz, 1991).

Any physical description involves many entities whose properties the theory has the task to describe. The standard description involves some absolute framework with respect to which properties are defined. In Newtonian physics, for instance, the background is a three dimensional Euclidean space and a one dimensional universal time. General relativity essentially is a background independent theory where the fundamental properties of the elementary entities consist entirely of relationships between those elementary entities. In 1912 Einstein had found the basic form of the gravitational field but it took him three years longer to write the equations of motion. His covariance principle required that the laws of nature were the same in all reference frames. But in a generally covariant theory statements of the kind of “what is the value of the gravitational field at coordinates x^a” make no sense. Indeed, a coordinate transformation can assign a region with large curvature to a coordinate point that prior had low curvature. In 1915 Einstein solved the problem. The idea is that it is only possible to describe relations. For example it is invariant to state that in a region in which certain light rays are present space–time has certain geometric properties (e.g. curvature). Einstein himself put is this way: “the results of our measuring are

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http://dx.doi.org/10.1016/j.shpsb.2013.09.010

nothing but verifications of… meetings of the material points of our measuring instrument with other material points, coincidences between the hands of a clock and points on the clock dial and observed point events happening at the same point at the same time." In our view this relational vision of background independence is the main guiding principle that must be followed when constructing a theory of quantum gravity. In such a theory only observable quantities (that are invariant under general coordinate transformations) can be associated with physical quantum operators. In the last few years there has been important progress in the description of the evolution and geometry in terms of such quantities (Gambini, Porto, Pullin, & Torterolo, 2009).

In the canonical approach, diffeomorphism invariance is reflected in the algebra of constraints. But this is not enough. In particular one has to pay careful attention to modifications that the theory may suffer through the use of non-traditional measures that arise in loop quantum gravity (Ashtekar & Lewandowski, 1997). We will see that this may restrict the types of diffeomorphisms that are recovered in the low energy limit of the theory. The non-traditional measures arise directly as a consequence of diffeomorphism invariance and are fairly unique (Lewandowski, Okołów, Sahlmann, & Thiemann, 2006; Fleischhack, 2006).

Related to the aforementioned principle is the second one: one should properly implement the dynamics of the theory. Since general relativity is a generally covariant theory, the Hamiltonian vanishes and one is just left with a set of constraints from which the dynamics needs to be disentangled. The constraints satisfy an algebra that needs to be implemented at a quantum level. Enforcing the constraint algebra assures that the canonical framework, which splits space–time into space and time, represents a space–time diffeomorphism invariant theory (Teitelboim, 1973; Hojman, Kuchař, & Teitelboim, 1976). This poses tight constraints on the quantization process that otherwise contains a large degree of ambiguity. In particular if one uses lattices to regularize the theory, reproducing the algebra of constraints can become quite a challenge.

The last principle is local Lorentz invariance. What is meant by this in the context of canonical quantum gravity is that if one studies the low energy limit, the resulting graviton (and other particles if one couples the theory to matter) should have propagators that deviate from Lorentz invariance at most only slightly. We will illustrate with a calculation what is meant by "slightly" in this context. In particular, deviations from Lorentz invariance that become large at the Planck scale are unacceptable as was argued by Collins, Perez, Sudarsky, Urrutia, and Vucetich (2004).

We will provide examples of the three principles in action in the following sections.

2. Diffeomorphism invariance

The first guiding principle is diffeomorphism invariance, or to put it in other terms, background independence. Most physicists believe a modern theory of gravity should not depend on background structures, since then one would have to motivate where the structures came from, and the whole point of general relativity was to eliminate any preferred observers in nature.

In canonical gravity one uses a $3+1$ dimensional split to formulate the equations of the theory. That split, obviously, violates space–time diffeomorphism invariance. The resulting framework is still invariant under spatial diffeomorphisms, such symmetry being reflected in the presence of the diffeomorphism constraint. Spatial diffeomorphism invariance plays a key role in loop quantum gravity. It essentially determines the kinematical structure of the theory through the selection of an inner product that is unconventional from the point of view of ordinary field theories (Lewandowski et al., 2006; Fleischhack, 2006). In turn, this structure implies that physical operators, like those representing areas and volumes, have discrete spectra (Rovelli & Smolin, 1995).

The breakage of space–time diffeomorphisms only means that the equations are not invariant, the resulting theory still is. In fact, the algebra of constraints is known to enforce that the resulting formalism is space–time diffeomorphism invariant (Teitelboim, 1973; Hojman et al., 1976). So, in principle, if upon quantization one ended up with a set of operators representing constraints that under commutators close an algebra isomorphic to the classical one under Poisson brackets, one could be confident that the resulting quantum theory is space–time diffeomorphism invariant.

But as we mentioned, one faces difficulties in implementing the constraint algebra at a quantum level. Up to present, no models have met such requirement (loop quantum cosmology, where there are no spatial degrees of freedom, implements them trivially so it is really not a strong guiding principle for those models). Moreover, it is customary to propose to deal with the diffeomorphism and Hamiltonian constraints separately. The diffeomorphism constraint is solved via the group averaging technique (Ashtekar, Lewandowski, Marolf, Mourao, & Thiemann, 1995; Giulini & Marolf, 1999), a procedure that cannot be implemented for the Hamiltonian constraint. Treating the constraints differently raises the possibility that space–time diffeomorphism invariance will be violated.

One way to deal with the problem is to gauge fix the theory, eliminating some or all the constraints. Classically, a gauge fixed theory is by definition diffeomorphism invariant. Although it is not manifestly diffeomorphism invariant, since one is dealing with the theory in a form that has no gauge symmetries, the results are diffeomorphism invariant in the sense that they can later be translated into any gauge in terms of gauge dependent variables.

But upon quantization, even in gauge fixed scenarios, there are subtleties. For instance, it can happen that the resulting variables that appear in the models have different ranges of values than those in the classical theory. That can imply that the set of diffeomorphisms considered is a restricted one.

An example of this is present in the treatment of the exterior of a vacuum black hole space–time we discussed in Gambini and Pullin (2009a,b). In that case, one can gauge fix the variables to spherical symmetry. One is left with two canonical pairs, one "longitudinal" along the radial direction E^x, A_x and a "transverse" one E^φ, A_φ, with the variables depending on the radial coordinate x and time t. One can further gauge fix the radial variable so that the diffeomorphism constraint is gone. The resulting Hamiltonian constraint is

$$H = -\frac{E^\varphi}{(x+a)\gamma^2}\left(\frac{A_\varphi^2(x+a)}{8}\right)' - \frac{E^\varphi}{2(x+a)} + \frac{3(x+a)}{2E^\varphi} + (x+a)^2\left(\frac{1}{E^\varphi}\right)' = 0, \tag{1}$$

where a is a constant and γ is the Barbero–Immirzi parameter. Multiplying by $2(x+a)/E^\varphi$ and grouping terms as,

$$H = \left(\frac{(x+a)^3}{(E^\varphi)^2}\right)' - 1 - \frac{1}{4\gamma^2}((x+a)A_\varphi^2)' = 0, \tag{2}$$

yields an Abelian constraint. Since the constraint is a total derivative, it can immediately be integrated to yield,

$$\int H\,dx = C = \left(\frac{(x+a)^3}{(E^\varphi)^2}\right) - x - \frac{1}{4\gamma^2}((x+a)A_\varphi^2), \tag{3}$$

with C being a constant of integration. At $x=0$ one can impose isolated horizon boundary conditions, which imply $1/E^\varphi = 0$ and