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The incongruent correspondence: Seven non-classical years of old quantum theory

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ABSTRACT

The Correspondence Principle (CP) of old quantum theory is commonly considered to be the requirement that quantum and classical theories converge in their empirical predictions in the appropriate asymptotic limit. That perception has persisted despite the fact that Bohr and other early proponents of CP clearly did not intend it as a mere requirement, and despite much recent historical work. In this paper, I build on this work by first giving an explicit formulation to the mentioned asymptotic requirement (which I shall call the Congruence Requirement (CR)) and then discussing various possible formulations of CP for emission on the basis of the primary literature as well as general physical and metaphysical considerations. I shall then show that, in all of the most probable interpretations of CP that consider quantum theory as a universal theory, any system incorporating both CR and CP for emission would in fact be inconsistent. Old quantum theory measurably contradicts classical physics *in the classical regime.*

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1. Introduction

1.1. *Congruence of frequencies in the low-frequency, high-quantum number regime*

The hallmark of old quantum theory is the assumption that physical systems can only exist in discrete “stationary states” and jump between these states with a certain probability. In line with this, Bohr (1913) famously assumed that only a subset of classically possible trajectories is available for the electron orbiting the nucleus. These “allowed states” are distinguished from the excluded ones by their particular energy (or angular momentum) content, which is always restricted to a sequence E_n (or L_n), ranging over a natural number n. These restrictions are called the quantization conditions. It is further assumed that when the electron is present in one such otherwise classical orbit, it does not emit radiation. Radiation is only produced when the electron jumps to an orbit with lower energy. If the energy of the initial orbit is E_n and that of the final orbit is E_m, the light radiated will be of frequency

$$\omega_{nm} = \frac{1}{\hbar}(E_n - E_m), \tag{1}$$

where $\hbar$ is Planck's constant. Bohr showed that the assumption $E_n = K/n^2$, where K is the Rydberg constant, suffices to explain the basic structure of the hydrogen spectrum.

There are two aspects of this theory that would be particularly shocking to any physicist of the time. First of all, the frequency of radiation bears no simple relationship to the mechanical frequency of the motion of the source at the time of radiation. This defies classical intuitions, according to which the frequency of a wave (whether mechanical or electromagnetic) is solely determined by the vibrational frequency of the source. Secondly, the theory makes no predictions about the intensity or polarization of the light radiated. The solution to these two problems turned out to be closely related. Let $\tau := n - m$ be the number of orbits the electron “jumps”. On the one hand, the orbital frequency decreases as n increases, and on the other, the radiation frequency increases with τ. Using these two facts, Bohr showed that in the limit in which $n, m \gg \tau$, where all frequencies are sufficiently small, we can get the radiation frequency to be equal to an integer multiple of the

Abbreviations: CP, Correspondence Principle; CR, Congruence Requirement; LFHQ, Low-Frequency, High-Quantum Number; HFHQ, High-Frequency, High-Quantum Number

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mechanical frequency:

$$\omega_{nm} \approx \tau\omega \tag{2}$$

where $\omega = \omega_n \approx \omega_m$. Although the early Bohr (1913, p. 13; 1920, pp. 429–430) tended to *stipulate* Eq. (2) and use that to derive K for the energy spectrum, $E_n = K/n^2$, it was later well-known that one can prove Eq. (2) quite generally from the quantization conditions and the condition that $n, m \gg \tau$ (Van Vleck, 1924, footnote 4).[1] This coincidence of the radiation frequencies with integer multiples of orbital frequencies in this limit provides much consolation, given that according to classical physics, any multiply periodic motion of principal frequency ω can be expanded in terms of its Fourier harmonics

$$x(t) = \sum_{\tau=0}^{\infty} C_\tau \cos(\tau\omega t + \alpha_\tau) \tag{3}$$

and will radiate all of its harmonics $\tau\omega$.[2] Let us call the regime in which Eqs. (2) and (3) are correct the low-frequency, high-quantum number (LFHQ) limit. Of course, Eq. (3) is never exactly true for a charged particle in the classical theory, because as the particle radiates, it loses some of its energy and thus cannot stay on the periodic path. However, Eq. (3) will be a good approximation for appropriately low frequencies.

1.2. Congruence of intensities in the low-frequency, high-quantum number regime

If each radiated frequency is related to one of the harmonics in the Fourier expansion of the electron's orbit, shouldn't the probability with which a given transition $n \rightarrow n-\tau$ occurs also be related to the relative weight of the τth harmonic in the series? One could formulate this as an educated guess, but as long as we are working in the LFHQ limit, this result follows simply from the requirement that quantum predictions go over to the classical ones in the regime in question. To see this, let us first ask what classical electrodynamics predicts about intensities.[3] Classically, when a charged particle undergoes a motion slow enough to be described by (3), it radiates all and only the harmonics present in the expansion and does so in accordance with Larmor's formula for power radiated:

$$P(t) = \frac{2e^2}{3c^3}\ddot{x}^2(t).$$

A simple calculation shows (Fedak & Prentis, 2002, pp. 338–339) that the average radiated power associated with the τth harmonic is

$$P_\tau = \frac{e^2}{3c^3}\tau^4\omega^4 C_\tau^2 \tag{4}$$

Now all we need to add in order to find quantum transition probabilities is the assumption that the observational predictions of quantum mechanics be in congruence with those of classical electrodynamics in the limit where the predictions of the latter have been confirmed.

> Congruence Requirement (CR): The statistical observable predictions of any quantum theory must be in agreement with those of classical physics within the margin of error in the limit where classical physics has been tested and confirmed.

To be sure, "the limit where classical physics has been tested and confirmed" is in need of clarification. Indeed, any theory-independent formulation of CR leaves open the question as to what range of what theoretical variables should be taken to represent this classical limit in any particular theory. In the case of old quantum theory, limits such as high quantum numbers, large numbers of quanta, large distances, large masses, and characteristic actions much larger than $\hbar$ might come to mind. My arguments below turn out to hold up under all mentioned definitions of "the classical limit". It is important to note, however, that low mechanical frequency is not a reasonable criterion for the classical realm. Synchrotron radiation, systems of radiating binary stars, and in general macroscopic radiating systems do not require quantum treatment in modes of high frequency of revolution. And of course a fundamental challenge is to explain in comparison to what the "low frequencies" are supposed to be low.[4]

What does CR teach us about transition probabilities? First note that the mechanism of radiation is very different in the quantum and classical pictures. While in the classical theory all harmonics are radiated simultaneously, quantum mechanics dictates that the electron in the orbit $x(t)$ has only one chance of emitting radiation by falling into a lower orbit, thus radiating one of the harmonics. If these transitions occur with the right probabilities, however, an *ensemble* of particles will emit a spectrum identical to what is expected from a classical ensemble (hence the "statistical" qualification). In order for this to occur we would need to have

$$P_\tau = A_{nm}(\hbar\omega_{nm}) = A_{nm}(\hbar\tau\omega),$$

where A_{nm} is defined as the rate per unit time of the quantum transition $n \rightarrow m$ accompanied by a radiation of frequency $\tau\omega$ in the LFHQ limit. This results in

$$A_{nm} = \frac{e^2}{3c^3\hbar}\tau^3\omega^3 C_\tau^2. \tag{5}$$

CR dictates that any proposed formula for quantum transition probabilities must converge to the right-hand side of (5) in the LFHQ limit.

Thus, we have derived a precise asymptotic relationship between quantum mechanical amplitudes and their classical counterparts using merely the laws of classical electrodynamics, the Congruence Requirement, and the quantization conditions.[5] In the following section, I will show how this formula serves as a segue to various formulations of the Correspondence Principle for emission.

2. On the search for a Correspondence Principle

There can be little doubt that what the early Bohr had in mind with "the Correspondence Principle" was intended to go beyond a congruence requirement. In a conversation with Rosenfeld, a frustrated Bohr is reported to have said that "the requirement that the quantum theory should go over to the classical description

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[1] The proof invokes action-angle canonical coordinates and the fact that $\omega_k = \partial W/\partial J_k$ where W is the energy of the system and J is the "action" variable (k=1, 2, 3 for the three spatial coordinates). Given the quantization condition $J = n\hbar$ and the assumption expressed in the equation $n, m \gg \tau$, we can write the energy released in a transition as $\hbar\omega_{nm} = \Delta W \approx dW = \sum_k(\partial W/\partial J_k)dJ_k = \sum_k \omega_k dJ_k \approx \sum_k \omega_k \tau\hbar$, which gives $\omega_{nm} = \sum_k \tau\omega_k$. Going from ΔW to dW and from dJ_k to $\tau\hbar$ are valid moves only when the consecutive orbits are infinitesimally close to each other, as would be the case in the LFHQ limit.

[2] Throughout this paper, I shall work with the case of a one-dimensional system. For a general three-dimensional system, $\tau\omega$ must be replaced with $\tau_1\omega_1 + \tau_2\omega_2 + \tau_3\omega_3$ and the sums would be over all τ_is. My conclusions will not be affected by this simplification. Also note that the inclusion of negative τs is a matter of regrouping of terms and does not change the physics.

[3] Similar considerations apply to the question of polarizations, which I shall not discuss.

[4] This discussion should not be confused with the discussion of the criterion that the de Broglie wavelength be short compared to the distance over which potential energy is considerable.

[5] The quantization conditions were employed in the proof of (2). See footnote 1.