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A conceptual discussion on electromagnetic units – Extending mechanical units towards a global system of units

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ABSTRACT

A comparative review of the different systems of units that are most usual in electromagnetism leads to the proposal of a new system of units. In this system, the gravitational constant acquires the role of an interaction constant, both for gravitational and electromagnetic interaction, as a result of a redefinition of electric charge. In this way, the new system of units extends in a natural manner to mechanics. The comparison between the gravitational and electromagnetic interactions is of particular relevance.

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1. Introduction

As is well known, the problem of the units and dimensions in electromagnetism has given rise to many debates in the past, as stated by J. D. Jackson (Jackson, 1975). Although it is an accepted matter and it seems to lead to no conceptual problems nowadays, it is indeed surprising that the International System of Units MKSA introduces a new unit, the ampere (A), which nowadays is defined by means of a "mechanical" experiment. This means, as Jackson (Jackson, 1975) also states, that the ampere is actually a derived unit. Now, if it is a derived unit, why it is not introduced as such a derived unit, as e.g. the newton or the joule are? The reason is that for historical reasons, the coulomb (C) had been introduced before, fixing the values of some given constants appearing in its definition, Coulomb's law. But why has this historical mess not been mended? We do not know exactly why, but the probable reason is the deeply rooted approach that it is enough to recognize the problem, without a need to find a reasonable solution. It is not surprising to find comments in course books stating that the present units of measurement are an obstacle when teaching electromagnetism – at least from a conceptual point of view –, as reflected in different ways in the references (Jackson, 1975; Reitz, Milford, & Christy, 1996). Other authors (Tipler & Mosca, 2008, and Sears, 1951) prefer to make no mention of it and ignore the problem (maybe correctly from a didactic viewpoint).

In our view, the International System of Units fulfills two different functions which sometimes are mixed up:

1. It sets some base units of measure or standards, establishing a connection with real objects or experiments which can be carried out *independently of the Physics laws*. These standards are supposed to be *taken out directly from reality*. This is the case of the definitions of second, meter, kilogram, and also ampere.
2. It recommends some given units for the different physical magnitudes as defined by theory, either base or derived magnitudes, such as length, speed, gravitational mass, force, work, etc.

It is not our purpose to argue about the standards which support the base units. But we do think that the choice of the units used affects the way how we interpret the physical formulas. We think that they can also change the underlying concepts that allow the development of Physics theories, leading to theoretical results which are either already known or seen from another point of view, or even new. Therefore, it is worth choosing those units with which Physics expressions adopt especially simple forms, according to some well defined criterion.

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As explained by Guissard (1972), whenever we have a physical law, for example $\vec{F} = m\vec{a}$, we always can multiply both equation members by the same constant, $K\vec{F} = Km\vec{a}$, and redefine some of the involved magnitudes, e.g. $\vec{F}^* = KF$; $m^* = Km$, so that, formally, the same relationship $\vec{F}^* = m^*\vec{a}$ is fulfilled. The new magnitudes will then establish a correspondence with the same standards through some dimensions which will be different or not from the original ones, depending on the units that the constant K has. If K has no units, no change of dimensions will occur. But if K has dimensions, we will have a change of dimensions. Anyway, a change of dimensions never means a change of base units or standards.

This procedure will be allowed in so far as it helps to understand or to make the Physics expressions simpler. On the contrary, it will be unwise if it leads to introducing units which do not match the problem. The different systems of units, especially the electromagnetic ones, are based on this kind of transformations.

In our opinion, the current definition of the ampere is actually a standard. The standard 1 A consists of having a given charge flow that causes the specified force in newton when carrying out the experiment stated in its definition. In order to have 1 A, the experiment has just to be carried out. No physical law is needed! If at all, the ampere is being debated as a standard, as at present the kilogram is being debated (Steiner, Williams, Newell, & Liu, 2005) and as earlier the meter was debated too.

A different problem is, as explained, the choice of the appropriate dimensions, once we have got the standard. The possibility of defining the ampere as a derived unit is no novelty (Guissard, 1972). The important issue is how to come to a concrete proposal and *which its conceptual contribution is*.

All this mess with the electromagnetic units contrasts with the almost total lack of debate for the units used in gravitation and in mechanics in general, when electromagnetic forces are not present. In fact, it is surprising that the gravitation theory does not use any new unit with respect to the units used in mechanics (if electromagnetic forces are not present). Everybody agrees with the way the equations for the gravitational field and the equation of motion for particles are written, at least within the non-relativistic theory. On the contrary, there are a couple of ways to write the Maxwell–Lorentz equations and there is very little agreement about their non-relativistic limit.[1]

We actually search for a global system of units. As such a global system, we understand a system of units that allows to implement the *quantum→classical* limit and/or the *relativistic→non-relativistic* limit without ambiguities. We will not consider the so-called natural systems of units (Planck, 1899) based on setting the value of some "universal constants" as a unit, so that these do not appear in the formulas any more (Maksymowicz, 1976; Barlett, 1974). For example, if suitable units are used, the value of the Plank constant can be set as $\hbar = 1$ only if the quantum theory is being used and there is no intention to take the step to the *classical limit*, i.e. to set $\hbar \to 0$.

The same occurs with the relativistic constant c. If we are interested in analyzing the transition from the relativistic to the non-relativistic theory, i.e. $c \to \infty$, we cannot take $c = 1$.

It has to be specified what is being understood as non-relativistic limit, distinguishing it from the particle low-speed limit. We can take a relativistic expression and see which consequences arise in case that particles move more or less slowly. But if we do $c \to \infty$, we are transforming this whole theory into another different one. The relativistic theory has a fundamental constant, c, whose value can be fixed in one way or another, depending on the units used. The non-relativistic theory does not have this constant.

From the conceptual point of view, it is important to distinguish between (1) carrying out the approximation of theory to a specific experimental situation and (2) going from a complete theory to another complete theory which is already contained in the first one for a specific value of some of its parameters (in our case, the constants c and $\hbar$). In the first case, the approximation consists in imposing restrictions like low speeds, weak fields, etc., because this is fulfilled in the specific experimental situation. In the second case, this is what happens with the grand theories in Physics. Indeed, Newtonian mechanics is a well established theory, with a level of internal consistency which is comparable to that of relativistic mechanics or quantum mechanics. The latter ones are theories with more parameters (c and $\hbar$, respectively). For a given value of these parameters, we have to retrieve the Newtonian theory completely.

This hierarchy between Physics theories is well known. Only when the theories have a sufficient degree of development, it is possible to analyze the role played by their different pieces. This is what has happened (and still happens) with the constant c, which characterizes the theory of relativity, although it was identified and used in electromagnetism, for historical reasons, long before relativity was born. In the development of electromagnetism, before Einstein, the really important thing was the construction of a theory which could account for the measurable electromagnetic phenomena. Nobody was conscious that, with this development, a logical contradiction was being created, which, as we know, Einstein resolved with the special relativity.

When a theory reaches its limit of experimental validity, another one has to be found as an extension of that old theory, but this does not mean at all that the old theory dies. An experimentally displaced theory may have a sound logical and conceptual structure on its own, so that it may be interesting to continue its development as a help to structure the new theories. This has been the case with Newtonian mechanics and its development in the form of analytical mechanics. This "classical" development has been absolutely decisive when developing both relativistic mechanics and quantum mechanics, although historically, the bulk of the analytical mechanics was created long before the beginning of the relativity and quantum theories.

This seems to be absolutely clear when we deal with quantum theories and take the step to classical theories, but it does not seem so clear in the case of non-relativistic limits in electromagnetism. If somebody notices the presence of Planck constant $\hbar$ in any expression concerning any phenomenon, he or she will undoubtedly qualify the phenomenon as a quantum phenomenon. If we assure him or her that the phenomenon is not due to any quantum effect, he or she will probably say that the expression in question is "poorly written". This explains why the usual systems of units (apart from the aforementioned natural systems of units; see Section 3) do not use the Planck constant in their definition. So, in general, taking the classical limit means taking the limit $\hbar \to 0$ in the involved quantum theory.

On the contrary, in electromagnetism it is relatively usual to see expressions containing the constant c and describing phenomena which usually are qualified as non-relativistic (for example, the magnetic term of the Lorenz's force is written as $(1/c)\vec{v} \times \vec{B}$ in some systems of units; also in the International System of Units, Gauss's law is written as $\vec{\nabla} \cdot \vec{E} = 10^{-7}c^2\rho$).

In a paper published recently (Jaén & Molina, 2013), one of the authors used the Maxwell–Lorentz equations to infer a possible form of the Newtonian equations of the gravitational field which are nearer to the relativistic equations. It was not a paper dealing with electromagnetism, but we think that the units used in the

[1] More specifically, there is little agreement about the theories invariant under the Galileo group which can be constructed implementing different limits on the basis of Eqs. (5) and (7). See reference (Le Blanc & Levy-Leblond, 1973).