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# Stability analysis of an aeroelastic system with friction

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Stability

**Abstract** In this paper, harmonic balance method, exact formulation and numerical simulation method are adopted to study the effects of different friction stiffness on the stability of 1.5 degrees of freedom aeroelastic system. On this basis, the expressions of input energy and dissipated energy are deduced, and the energy method is used to reveal the mechanisms of the stable boundary and unstable boundary existing in the system and the effects of different friction stiffness on the stability of the system. Studies have shown that the stability region and the critical aerodynamic damping ratio of the system rise with the increase of the friction stiffness, while the friction stiffness has little effect on the stability boundary. In the analysis of the stability of system, the results of harmonic balance method, exact formulation and Newmark of numerical simulation method are in good agreement. Compared with exact formulation and numerical simulation method, the concept and conclusion of harmonic balance method are simple in the system stability analysis.

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## 1. Introduction

The phenomenon of friction is widespread in mechanical structure. The problem of structural response with friction has been studied by many scholars. Ding<sup>1</sup> overviewed ten approximate friction models including Coulomb friction, Bristle friction model and Reset Integrator friction model, and made a wonderful comment about the characteristics and range of applications of these friction model. Den Hartog<sup>2</sup> obtained the exact solution of a single degree of freedom model with Coulomb friction firstly. Yeh<sup>3</sup> extended the exact solution to two degrees

of freedom model. Griffin<sup>4</sup> adopted a hysteretic spring friction model while studying the response of the blade with friction damper, and simplified the blade system to a single degree of freedom of mass-spring model, then discussed the maximum response of the system by Ritz method. Hao and Zhu<sup>5</sup> proposed a method to calculate the response of complex structure with friction damping – dynamic compliance method, and the method was applied to calculate the steady-state response of a turbine blade which included a friction damper. For these hysteresis non-linear problems, Ding and Chen<sup>6</sup> studied the main resonance of a class of bilinear hysteresis nonlinear non-autonomous system with self-excited features, and revealed the relationship between the response and system parameters by singularity theory. Sinha and Griffin<sup>7</sup> studied the flutter of a 1.5 degrees of freedom aeroelastic system with hysteretic spring friction based on the research of Griffin by using the negative damping force to represent the unsteady aerodynamic force, and used harmonic balance method to discuss stability problems of the system with small friction stiffness. The conclusion of the research has been verified by a numerical simulation

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method. Mignolet et al.<sup>8-10</sup> also researched the model studied by Sinha in the study of the effects of frictional structural non-linearity on the system in the presence of negative aerodynamic damping, and studied the stability of the system with a small friction stiffness through harmonic balance method, exact formulation and numerical simulation method. Li et al.<sup>11,12</sup> solved the dynamical equation of a two-dimensional airfoil with polynomial hysteresis nonlinearity in pitch using numerical integration method and harmonic balance method respectively.

In this paper, further study about the effects of different friction stiffness on the stability of the system is performed based on the previous research model and research methods, and the mechanism is revealed through the energy method. The reliability problems of these three analysis techniques in the solution process are also discussed, which provide a reference for selecting analysis methods.

## 2. Model introduction

Griffin<sup>4</sup> derived the equations of motion of the blade detailed in the study of how to use the blade-to-ground damper to suppress the gas turbine blade resonance. Griffin used the first-order mode as the general basic vector in the equations of motion due to the first-order mode predominates while the blade is resonating, which made the infinite degrees of freedom model degenerate into a single degree of freedom model. The system included a hysteretic spring friction model which introduces a dependent coordinate system; therefore it was defined as 1.5 degrees of freedom system.

Sinha and Griffin<sup>7</sup> researched a 1.5 degrees of freedom system in the study of effects of friction damping on flutter of gas turbine blades, and made some results in the research of system dynamic stability by using the negative damping force to denote the unsteady aerodynamic force. Mignolet et al.<sup>8-10</sup> also researched the 1.5 degrees of freedom system in study of the effects of frictional structural nonlinearity on the system in the presence of negative aerodynamic damping.

The object of the above study is a non-linear aeroelastic system. Because of the lack of accurate and reliable calculation, analytical methods and test specifications, and also the lack of complete understanding of the mechanism about complex nonlinear aeroelastic phenomena, for the nonlinear aeroelastic system, the main solutions for nonlinear aeroelastic problems are to try to decrease the non-linear level, increase the safety factor, or use linear analysis tools for processing in the current engineering. This way is effective for some weak non-linear problems, but the way which is to improve the safety margin to ensure flight safety is contrary to the optimization of reducing the structural weight to improve the efficiency of the structure. For strong non-linear factors, the linear methods can still not predict the aeroelastic response and stability accurately. Taking into account the role of the expansion of the flight attitude and scope, the widespread use of advanced materials, the enhancement of flight control system and other factors in the current and future aircraft design, it is urgent to study the non-linear aeroelastic analysis and applied research for guaranteeing flight safety and improving aircraft performance.

There are some advantages to analyze the 1.5 degrees of freedom system. In the first place it can avoid solving the cumbersome nonlinear equations. Secondly the regularity charac-

terized by the system can be assured not to lose generality. The simplified model of the 1.5 degrees of freedom is shown in Fig. 1, and the motion equation of the system is

$$m\ddot{u} + c\dot{u} + (k + k_d)u = Q \cos \bar{\omega}t + k_d z \tag{1}$$

The system introduces a hysteretic spring friction model, the maximum static friction is  $F_d$ , the spring represents the coefficient of shear elasticity between two frictional surfaces, and  $k_d$  is notated as the friction stiffness here. If  $|u - z| < F_d/k_d$ , the model is in the viscous phase; when  $|u - z| = F_d/k_d$ , slid occurs, and the friction is  $F_d$  at this time.

For the nondimensionalization of Eq. (1), let  $\varepsilon = k_d/(k_d + k)$ ,  $\xi = -c/\sqrt{4m(k_d + k)}$ ,  $x = u/u_0$ ,  $y = z/u_0$ ,  $t = \bar{t}/T$ ,  $\tilde{\omega} = \bar{\omega}T$ ,  $f_0 = Q/F_d = Q/(k_d u_0)$ . where

$$u_0 = F_d/k_d, \quad T = \sqrt{m/(k_d + k)}$$

Then Eq. (1) can be expressed as

$$\ddot{x} - 2\xi\dot{x} + x = \varepsilon y + \varepsilon f_0 \cos \tilde{\omega}t \tag{2}$$

Eq. (2) is the equation of motion of the system depicted in Fig. 2.  $\varepsilon$  is the friction stiffness, and the second term of the equation is the negative damping force ( $\xi > 0$ ), which represents the aerodynamic force of the 1.5 degrees of freedom system.

When  $f_0 = 0$ , the system depicted in Fig. 2 is the simplest of 1.5 degrees of freedom aeroelastic system. In this paper, the following analysis is based on  $f_0 = 0$ .

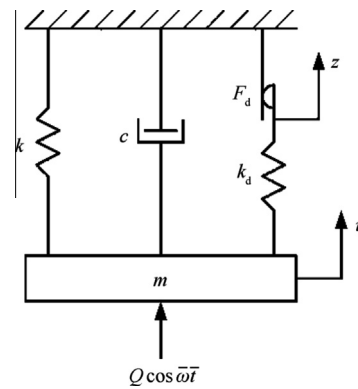


Fig. 1 Simplified model.

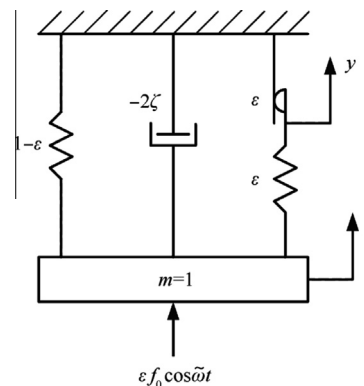


Fig. 2 Equivalent model.

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