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Dynamic stiffness vibration analysis of an elastically connected three-beam system

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Abstract

An exact dynamic stiffness method is developed for predicting the free vibration characteristics of a three-beam system, which is composed of three non-identical uniform beams of equal length connected by innumerable coupling springs and dashpots. The Bernoulli–Euler beam theory is used to define the beams' dynamic behaviors. The dynamic stiffness matrix is formulated from the general solutions of the basic governing differential equations of a three-beam element in damped free vibration. The derived dynamic stiffness matrix is then used in conjunction with the automated Muller root search algorithm to calculate the free vibration characteristics of the three-beam systems. The numerical results are obtained for two sets of the stiffnesses of springs and a large variety of interesting boundary conditions.

Keywords: Three-beam system; Bernoulli–Euler beam; Dynamic stiffness method; Free vibration

1. Introduction

Since beams comprise the basic components of so many modern structures, it is necessary for the design engineers to evaluate the dynamic characteristics of beam structures effectively. Preliminary dynamic analyses of the beam structures also help to optimize the design and avoid future investments on repairs. Due to the wide use of the beam-type structures in many branches of modern civil, mechanical, and aerospace engineering, the free and forced flexural vibrations of the single uniform beams with different support conditions have been studied extensively by many investigators. However, the dynamics of the built-up beam-type structures such as the elastically connected three-beam systems under investigation in this paper is still a subject of great interest to researchers. The physical model of the three-beam system is composed of three parallel, slender, prismatic and homogeneous beams connected each other by innumerable coupling springs and dashpots. A relatively few works have been done on this kind of structures. The three-beam system can be considered as an ideal model of structural elements practically used such as a complex continuous system consisting of three parallel one-dimensional structures joined by linear viscoelastic layers, or as an approximate model of the sandwich beams, or as a model which has the future applications. A literature survey shows that the different aspects concerning the dynamic characteristics of the elastically connected parallel-beam systems have been presented in a few works. Refs. [1–11] form a carefully selected sample of this literature.

Seelig and Hoppmann [1] presented the development and solution of the differential equations of motion of a system of elastically connected parallel beams. Vibration experiments were performed to determine the degree of applicability of the theory. The case of a double-beam system consisting of two identical beams elastically connected was studied in detail. Kessel [2] investigated the system consisting of two parallel, simply-supported beams that were elastically connected and subjected to a moving load

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that oscillated longitudinally about a fixed point along the length of one of the beams. Rao [3] derived the differential equations governing the flexural vibrations of the systems of elastically connected parallel beams with the effects of shear deformation and rotary inertia considered. For computational purposes, the simple case of a three-beam system with identical springs with all ends hinged was considered. Chonan [4] investigated the dynamical behaviors of two beams connected with a set of independent springs subjected to an impulsive load. He considered the case of a two-beam system consisting of two identical beams in the analysis. Hamada et al. [5] analyzed the free and forced vibrations of a system of two elastically connected parallel upper and lower beams having unequal masses and unequal flexural rigidities by using a generalized method of finite integral transformation and the Laplace transformation. Aida et al. [6] presented a beam-type dynamic vibration absorber which was composed of an absorbing beam under the same boundary conditions as the main beam and uniformly distributed, connecting spring and damper between the main beam and absorbing beam. Chen and Sheu [7] developed the dynamic shape functions and the dynamic stiffness matrix of a layered-beam element which was composed of two parallel beams of uniform properties with a flexible core in-between. Chen and Lin [8] presented the structural analysis and optimal design of a dynamic absorbing beam which was attached to the main beam with a viscoelastic layer or other mechanism of similar effect. Vu et al. [9] presented an exact method for solving the vibration of a double-beam system subject to harmonic excitation. The Bernoulli–Euler model was used for the transverse vibrations of beams. Oniszczuk [10] discussed the free transverse vibrations of two parallel simply-supported beams continuously joined by a Winkler elastic layer. The motion of the system was described by a homogeneous set of two partial differential equations, which was solved by using the classical Bernoulli–Fourier method. Oniszczuk [11] devoted to analyze the undamped forced transverse vibrations of an elastically connected double-beam system in the case of simply-supported beams. The classical modal expansion method was applied to determine the dynamic responses of the beams due to arbitrarily distributed continuous loads.

It can be seen from these references that most works only investigate the case of double-beam system and are limited to the case of some particular boundary conditions. Although few research papers can be found to deal with the elastically connected multi-beam system, these studies are limited to the particular case of identical beams. The general case of the three-beam system in which the geometry and material, as well as the boundary conditions of the three parallel beams of uniform properties are all different has not been investigated. The general vibration analysis of such an elastically connected three-beam system is complicated and laborious in view of a large variety of possible combinations of boundary conditions. This paper sets out to reveal the properties of the free vibration of a system of three elastically connected uniform beams having unequal masses and unequal flexural rigidities. The dynamic interaction of the three-beam system is emphasized.

The aim of this paper is to extend the library of dynamic stiffness matrix currently available in literature by determining the dynamic stiffness matrix for an elastically connected three-beam system. It appears that no one has used the dynamic stiffness method to predict the free vibration characteristics of the three-beam systems and to investigate the interaction occurring in three-beam systems. As it will be shown later, considerable analytical and computational efforts are required to derive the dynamic stiffness matrix of a three-beam system. Some advantages of the dynamic stiffness method are well known [12,13], particularly when higher frequencies and better accuracies are required. This method also forms a useful comparator when the conventional finite element or other approximate methods are used. It is expected that the theory presented here is more accurate for the slender three-beam systems due to the effects of shear deformation and rotary inertia neglected in the formulation. This paper appears to be the first to obtain the dynamic stiffness matrix formulation of the elastically connected three-beam systems. The dynamic stiffness matrix is formulated in detail from the general solutions of the basic governing differential equations of a three-beam element in damped free vibration. The application of the derived dynamic stiffness matrix to calculate the free vibration characteristics of the three-beam systems uses the automated Muller root search algorithm [14]. The influences of the stiffness of springs and boundary condition on the dynamic behaviors of the particular interesting examples are extensively studied.

2. Formulation of problem

The physical model of the three-beam system under consideration is composed of three parallel, slender, prismatic and homogeneous beams connected each other by innumerable coupling springs and dashpots, as shown in Fig. 1. All beams have the same length and the small amplitude vibration of the system is investigated.

If both the effects of shear deformation and rotary inertia of the beams are ignored, the kinetic and potential energies of the elastically connected three-beam system can be given by

$$T = \frac{1}{2}\int_0^L \rho_1 A_1 (\dot{w}_1)^2\,dx + \frac{1}{2}\int_0^L \rho_2 A_2 (\dot{w}_2)^2\,dx + \frac{1}{2}\int_0^L \rho_3 A_3 (\dot{w}_3)^2\,dx \tag{1a}$$

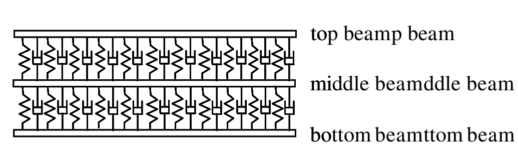

Fig. 1. Model of three-beam system.