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# Parameter selection in the Green's function parabolic equation

Jennifer L. Cooper *,[1], David C. Swanson

*Graduate Program in Acoustics, The Pennsylvania State University, P.O. Box 30, University Park, PA 16801, United States*

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**Abstract**

The accuracy of the Green's function parabolic equation (GFPE) has already been confirmed for outdoor sound propagation over flat ground with a slowly varying sound speed profile and/or atmospheric turbulence. However, use of parabolic equation methods for prediction is generally limited to experts because of their dependence on numerous algorithm parameters that have significant impact on the accuracy of prediction. The present work offers a set of GFPE parameters, outlined in Table 1, that will provide accurate results for a variety of physical situations involving atmospheric propagation. The guidelines are found by comparing GFPE results to analytical results for a variety of situations and parameter choices and noting which combinations of parameters lead to accurate (within 3 dB of the analytical solution) results and which do not. A significant source of error in GFPE results is inaccuracy in the starting field or inappropriate starting field selection. The selection criteria for starting field and other parameters are discussed here.


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* Corresponding author. Tel.: +1 703 321 4760; fax: +1 703 321 4609.
*E-mail address:* cooper.jennifer@ensco.com (J.L. Cooper).
[1] Present address: ENSCO Inc., 5400 Port Royal Road, Springfield, VA, United States.

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## 1. Introduction

The need for a greater understanding of the propagation of sound outdoors over a variety of terrains and in a range of weather conditions is widespread. This understanding could be applied for uses in the military, in the entertainment industry, in city planning, and in industrial noise abatement, to name just a few. Often the need is for a prediction of the received levels for a system in which received levels cannot be measured, as is the case for planned building projects that would affect the soundscape. It would be difficult, if not impossible, to develop an analytical propagation model capable of handling realistic sound speed and ground height profiles as well as range-dependent ground impedance, but a numerical model, such as a parabolic equation code, could accommodate such profiles. One such model, the Green's function parabolic equation (GFPE) [1] was developed primarily for outdoor sound propagation over ground of finite impedance.

## 2. Green's function parabolic equation

The GFPE avoids the problems associated with finite impedance ground that occur in other parabolic equation codes by finding three terms separately at each range step and then adding the terms together again before the next step. These three terms are the direct wave, the specularly reflected wave, and the surface wave. The ground has been assumed to be locally reacting, which was shown by Rasmussen [2] to be a valid assumption for most grass-covered grounds. This assumption simplifies the approximate analytical solution for the reflection coefficient and surface wave.

Assuming azimuthal symmetry and $kr \gg 1$, using cylindrical coordinates, neglecting backward traveling waves, and treating the wavenumber as a constant wavenumber plus a small perturbation,

$$k^2(z) = k_0^2 + (\delta k(z))^2, \tag{1}$$

Gilbert and Di [1] developed the following way to implement the Green's function parabolic equation:

$$\psi(r+\mathrm{d}r) = \mathrm{e}^{\left(\frac{j\mathrm{d}r(\delta k)^2}{2k_0}\right)}\left(\mathscr{F}^{-1}\left[(\tilde{\psi}(r,k') + R(k')\tilde{\psi}(r,-k'))\,\mathrm{e}^{\left(j\mathrm{d}r\left(\sqrt{k_0^2-k'^2}-k_0\right)\right)}\right]\right.$$
$$\left. + 2j\beta\tilde{\psi}(r,\beta)\,\mathrm{e}^{j\mathrm{d}r\left(\sqrt{k_0^2-\beta^2}-k_0\right)}\,\mathrm{e}^{-j\beta z}\right), \tag{2}$$

where $k(r,z) = \omega/c(r,z)$ is a wavenumber dependent on range and height, and $\psi = \sqrt{r}p$ is the pressure with the cylindrical spreading removed. $\beta = \frac{k_0}{Z_g}$ is the surface wave pole, $R(k) = \frac{kZ_g - k_0}{kZ_g + k_0}$ is the plane-wave reflection coefficient, $Z_g$ is the normalized ground impedance, and $\tilde{\psi}$ is the spatial Fourier transform of $\psi$. The inverse Fourier transform is indicated by $\mathscr{F}^{-1}$. The approach represented by Eq. (2) is *exact* for constant sound speed profiles because $\delta k(z)$ is zero.

In Eq. (2), the three terms discussed previously are evident. The first term is due to the direct path, the second term is due to the specularly reflected path, and the third term is the surface wave.