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Limited-memory scaled gradient projection methods for real-time image deconvolution in microscopy ☆,☆☆

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ABSTRACT

Gradient projection methods have given rise to effective tools for image deconvolution in several relevant areas, such as microscopy, medical imaging and astronomy. Due to the large scale of the optimization problems arising in nowadays imaging applications and to the growing request of real-time reconstructions, an interesting challenge to be faced consists in designing new acceleration techniques for the gradient schemes, able to preserve their simplicity and low computational cost of each iteration. In this work we propose an acceleration strategy for a state-of-the-art scaled gradient projection method for image deconvolution in microscopy. The acceleration idea is derived by adapting a step-length selection rule, recently introduced for limited-memory steepest descent methods in unconstrained optimization, to the special constrained optimization framework arising in image reconstruction. We describe how important issues related to the generalization of the step-length rule to the imaging optimization problem have been faced and we evaluate the improvements due to the acceleration strategy by numerical experiments on large-scale image deconvolution problems.

1. Introduction

Image deconvolution is a useful technique for improving the image quality of many types of microscope. Unfortunately, in case of large-scale imaging problems, the deconvolution process can require a too large computational time which leads to undesirable delay in the reconstruction process. This is mainly due to the slow convergence rate of the iterative deconvolution methods usually applied to microscopy images, such as the well-known scaled gradient minimization method called *Richardson–Lucy* (RL) *algorithm* [1,2]. To overcome this disadvantage, two strategies have been exploited in the last years: the acceleration of the deconvolution algorithms and the implementation of the algorithms on multiprocessor architectures, such as the graphics processing units (GPUs). The combination of the benefits from both of these strategies allowed to

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achieve promising time reductions in the deconvolution process, providing stimulus for further research in these fields. In this paper we focus on an iterative deconvolution algorithm that aims at exploiting a new step-length selection strategy for gradient descent methods to improve the convergence rate of state-of-the-art deconvolution approaches in microscopy. The considered step-length strategy has been recently proposed by R. Fletcher [3] in the context of limited-memory steepest descent methods for unconstrained minimization problems. In [3] numerical evidence has been also provided, indicating remarkable gain in the convergence rate over the classical Barzilai–Borwein (BB) step-length rule [4]. Since in the last years promising image reconstruction algorithms have been designed by exploiting BB-based rules within gradient methods [5–11], it is worthwhile to investigate if useful acceleration can be achieved with the new step-length selection idea. In particular, we focus on the algorithm for image deconvolution in microscopy provided by the *scaled gradient projection* (SGP) *method* recently developed in [12], that can be appropriately modified for managing the step-length rule proposed in [3]. SGP is a very general algorithm for minimization problems with simple constraints, able to exploit the scaled gradient directions as well as the selection rules for the step-length parameter. By combining an efficient step-length selection based on the adaptive alternation of the two BB rules [13,14] and a scaling strategy similar to that used by the RL algorithm, SGP has shown significant convergence rate improvements in comparison to RL, without excessive growth of the cost per iteration and/or loss of reconstruction accuracy [5]. These abilities allowed the GPU version of SGP, designed in [12], to show interesting performance as a real-time deconvolution algorithm. To efficiently equip SGP with the new step-length rule introduced in [3], crucial aspects concerning with the presence of scaled gradient directions and nonnegativity constraints need to be discussed. In this work we propose how to generalize the step-length rule to the SGP framework and provide numerical evidence of the time gain achievable in comparison to the BB-based step-length selection previously exploited by SGP.

The paper is organized as follows. In Section 2, the optimization problem arising in image deconvolution is stated and the RL and SGP iterative regularization algorithms are recalled; in section 3, the new step-length rule for SGP is introduced and, in Section 4, a computational study is presented for validating on large-scale image deconvolution test problems the SGP version equipped with the new rule. Finally, some conclusions and a discussion on possible future work are reported in Section 5.

2. The SGP method for image deconvolution

We shortly introduce the optimization problem arising from the *maximum likelihood* (ML) approach to image deconvolution in microscopy; for a deeper discussion of the image deconvolution problem, we refer the reader to [15,16]. Let us denote by $\boldsymbol{y} \in \mathbb{R}^n$ the detected blurred and noisy image and assume a linear model for describing the image acquisition process: $A\boldsymbol{x} + \boldsymbol{b}$, where $\boldsymbol{x} \in \mathbb{R}^n$ is the unknown object, $\boldsymbol{b} \in \mathbb{R}^n$ is the known positive background emission ($b_i > 0$) and A is the $n \times n$ imaging matrix representing the blurring phenomenon. The detected values y_i are nonnegative and the matrix A can be considered with nonnegative entries, generally dense and such that $\sum_j A_{ij} > 0 \ \forall i$ and $A^T\boldsymbol{e} = \boldsymbol{e}$, where $\boldsymbol{e} \in \mathbb{R}^n$ is a vector whose components are all equal to one. Furthermore, we may assume that periodic boundary conditions are imposed for the discretization of the Fredholm integral equation that models the image formation process, so that the matrix A is block-circulant with circulant blocks and the matrix–vector products $A\boldsymbol{x}$ can be done quickly, with $\mathcal{O}(n\log(n))$ complexity, by using the Fast Fourier Transform (FFT) [17]. If we assume that the detected values y_i are realizations of independent Poisson random variables, with unknown expected values $(A\boldsymbol{x} + \boldsymbol{b})_i$, the maximum likelihood approach to the deconvolution problem leads to the minimization of a suitable data-fidelity function called *generalized Kullback–Leibler* (KL) *divergence* (or *Csiszár I-divergence*) [18,19]:

$$f(\boldsymbol{x}) = \sum_{i=1}^{n} \left\{ y_i \ln \frac{y_i}{(A\boldsymbol{x} + \boldsymbol{b})_i} + (A\boldsymbol{x} + \boldsymbol{b})_i - y_i \right\}, \tag{1}$$

whose gradient and hessian are given by

$$\nabla f(\boldsymbol{x}) = A^T\boldsymbol{e} - A^T Z^{-1}\boldsymbol{y}, \tag{2}$$

$$\nabla^2 f(\boldsymbol{x}) = A^T Y Z^{-2} A, \tag{3}$$

where $Z = \mathrm{diag}(A\boldsymbol{x} + \boldsymbol{b})$ is a diagonal matrix with the entries of the vector $A\boldsymbol{x} + \boldsymbol{b}$ on the main diagonal and $Y = \mathrm{diag}(\boldsymbol{y})$. We observe that the hessian matrix is positive semidefinite in any point of the nonnegative orthant. Due to the ill-posedness of the image restoration problem [16], the matrix A could be very ill-conditioned and a solution of the convex optimization problem

$$\min_{\boldsymbol{x} \geqslant \boldsymbol{0}} f(\boldsymbol{x}) \tag{4}$$

does not provide sensible reconstructions of the unknown image. Two alternative strategies can be exploited to overcome this drawback. The first approach consists in looking for suited regularized reconstructions by early stopping iterative minimization methods applied to the problem (4). The second strategy requires to solve a regularized minimization problem

$$\min_{\boldsymbol{x} \geqslant \boldsymbol{0}} f(\boldsymbol{x}) + \beta f_R(\boldsymbol{x}), \tag{5}$$